# COMPUTER AIDED DESIGN OF LEAF SPRINGS

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# COMPUTER AIDED DESIGN OF LEAF SPRINGS

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DEDICATED

TO

MY TEACHERS

CERTIFICATE

Certified that, this work on 'Computer Aided Design of Leaf Springs' by Shri K. Shantharam in partial fulfilment of the requirements for the degree of Master of Technology of the Indian Institute of Technology, Kanpur, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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K. SHANTHARAM

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# NOMENCLATURE

| | | |
|---|---|---|
| a | - | Cantilever overhang (front). |
| b | - | Cantilever overhang (rear). |
| c | - | Spring index = D/d. |
| D | - | Pitch diameter (Chapter 1). |
| d | - | Wire diameter (Chapter 1). |
| e | - | Eye offset. |
| $e_{sz}$ | - | Size factor. |
| E | - | Young's modlus elasticity. |
| G | - | Rigidity modulus. |
| i | - | Number of coils, (Chapter 1). |
| $I_n$ | - | Moment of inertia. |
| K | - | Stress factor, (Chapter 1). |
| | - | Spring rate. |
| $K_a$ | - | Rate for front cantilever. |
| $K_b$ | - | Rate for rear cantilever. |
| $K_1$ | - | Buckling factor, (Chapter 1). |
| $l_o$ | - | Free length, (Chapter 1). |
| L | - | Length of spring. |
| $L_i'$ | - | Overhang of $i^{th}$ leaf. |
| n | - | Factor of safety, (Chapter 1). |
| | - | Number of full length leaves including mean leaf. |
| M | - | Bending moment. |
| N | - | Total number of leaves. |

$P$ - Load kgf.

$P_{cr}$ - Critical axial load, (Chapter 1).

$P_o$ - Spring scale = p/y, (Chapter 1).

$q_i$ - Curvature of $i^{th}$ leaf.

$r_c$ - Ride clearance.

$R$ - Radius of curvature.

$R_o$ - Radius of curvature (initial).

SF - Stiffenning factor.

$t$ - Thickness.

$t_{max}$ - Maximum thickness.

$V$ - Volume.

$w$ - Width.

$W$ - Weight.

$y$ - Total deflection, (Chapter 1).

$Y$ - Cantilever ratio = b/a.

$Z$ - Rate ratio of cantilever $(K_a/K_b)$.

$\delta$ - Deflection.

$\delta_a$ - Deflection of front cantilever end A.

$\delta_b$ - deflection of rear cantilever end B.

$\delta_s$ - Static deflection.

$\delta_{max}$ - Maximum deflection.

$\rho$ - Density.

$\sigma$ - Stress.

$\sigma_a$ - Safe stress or allowable stress,(Chapter 1).

$(\sigma_{asb})_i$ - Assembly stress of $i^{th}$ leaf.

$\sigma_{max}$ - Maximum allowable stress.

$\sigma_p$ - Load stress in any leaf.

## ABSTRACT

The design of a leaf spring is an important procedure for a mechanical design engineer. This design includes, the following parameters: thicknesses of leaves, lengths of leaves, camber of leaves and the stresses that the leaves undergo. The main stresses being assembly stresses and load stresses, the design of leaf radii, leaf cambers and leaf lengths become important. This design requires repetition of the same calculation for each leaf and until the desired stresses are obtained, the whole process is repeated over and again.

The present work is an attempt to develop an interactive package for the computer aided design of leaf springs. The program developed is interactive and the user has been given some control over a few parameters when the program is in execution mode. The user can arrive at a leaf combination to achieve the desired rate and stress distribution. Moreover, this program also provides for the design of a variable rate leaf spring of the helper type. This variable rate leaf spring design helps in designing springs for trucks and buses which operate at both zero and very heavy loads.

# CHAPTER - 1

## INTRODUCTION

### 1.1 Definition:

A spring is an elastic body whose primary function is to deflect or distort under load(or to absorb energy) and recover its original shape when the load is released.

A typical spring will have a linear load-deflection diagram as shown in Fig. 1.1, the material not being stressed beyond the elastic limit. The deflection is proportional to the load and will be true even if the acting load is a torque.

Fig. 1.2 shows two non-linear load-deflection diagrams. Curve A can be obtained with a thin flat circular plate or disk loaded to a large deflection. Curve B may result from an initially coned-disk (or Belleville) spring. The clock spring is another example which has a non-linear characteristic [1].

### 1.2 Functions of Springs:

Following are the few important functions of springs:

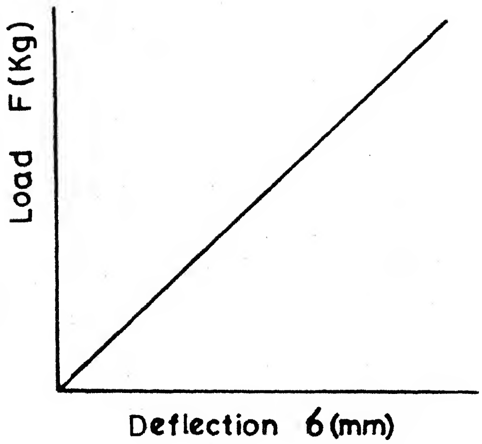



Fig.1.1 Linear load-deflection diagram of typical springs.

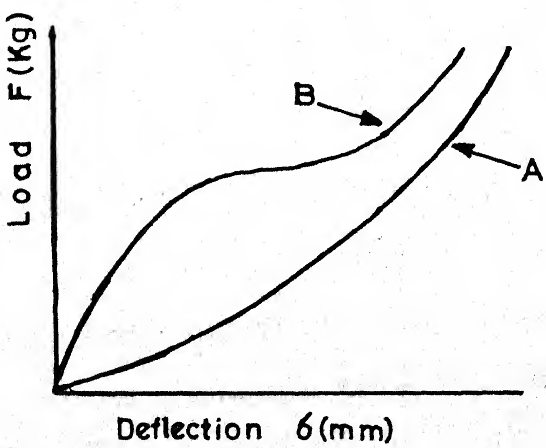



Fig.1.2 Nonlinear load-deflection curves of disk or Belleville springs.

a) To absorb or control energy and vibration due to shock as in car springs, railway buffers, spring supports and vibration dampers.

b) To control motion as in a cam and its follower, governor or valve, brake or clutch.

c) To store energy as in clockworks.

d) To measure forces as in spring balances, gages or engine indicators. [1,2].

## 1.3 Classification:

Springs are classified according to shape as indicated in Fig. 1.3. In Fig. 1.3a is shown a leaf spring and in Fig. 1.3b a helical conical spring. If the radius of the coils of a helical conical spring is constant it becomes a helical cylindrical spring as shown in Fig. 1.3c. If the angle of helix is zero, it becomes a spiral spring as in Fig. 1.3d. For large loads and deflections, disk springs can be used.

In leaf, spiral and disk springs, the stress induced by the load P, is bending. In a helical spring the main stress induced by an axial load P is torsion. If a helical spring is under torsion, the main stress is bending. [2,3].

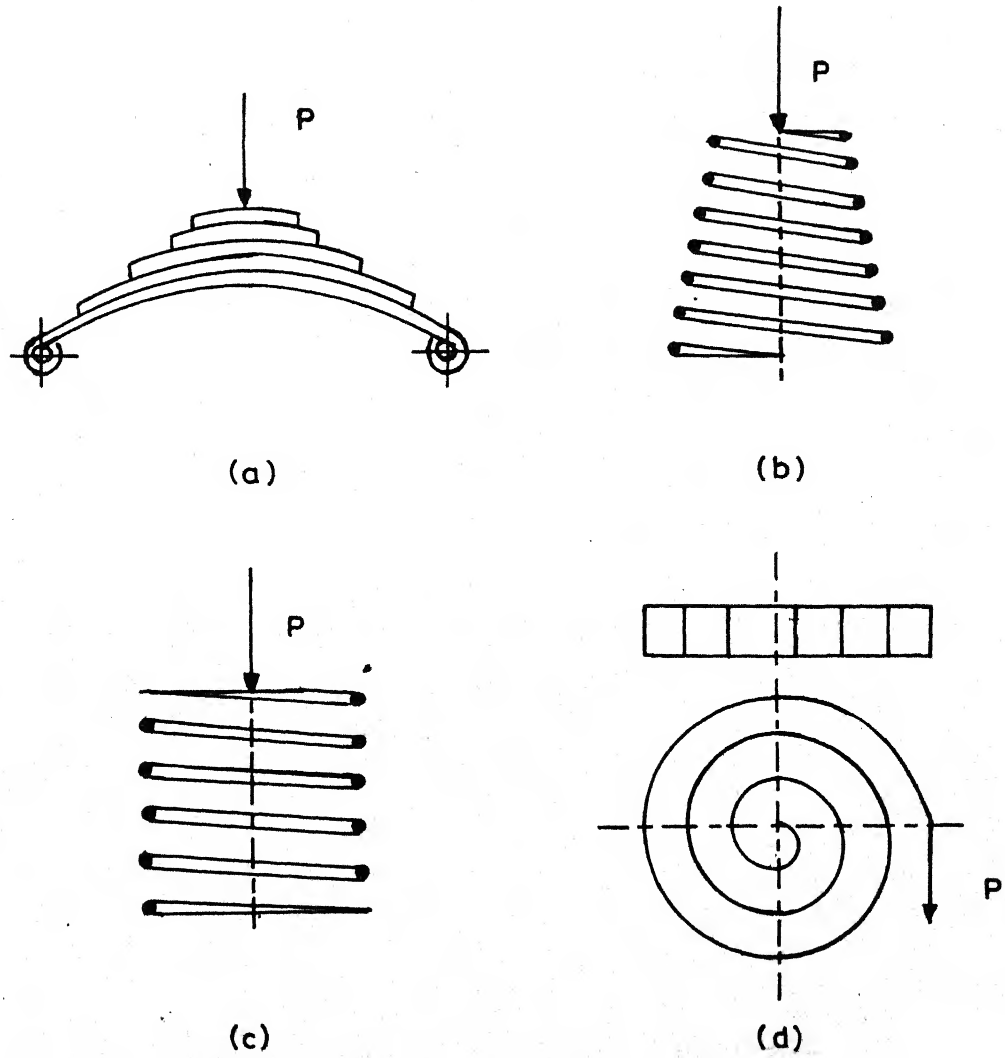



Fig.1.3 Shapes of springs.

## 1.4 Design of Helical Springs:

A very brief procedure for the design of helical springs is given in this section [2].

Given the load on the spring (P), and the desired spring scale ($P_o$) to design the spring, i.e. the pitch diameter (D), the wire diameter (d), the number of coils (i), and the free length ($l_o$).

First the pitch diameter is selected to conform to given space limitations or other conditions. The factor of safety (n) is next selected and the approximate value for the allowable stress ($\sigma_a$) is determined. The wire diameter can be found from

$$d = \sqrt[3]{\frac{8\,P\,D}{\pi\;\sigma_a}}$$

Based on this wire diameter, the size factor ($e_{sz}$) can be found from,

$$e_{sz} = 0.86 + \frac{0.07}{d}$$

This value can be used to find more accurate value of the allowable stress. The stress factor K can be determined from Fig. 1.4 [4] or from the equation,

$$K = \frac{40 - 1}{40 - 4} + \frac{0.615}{C}$$

where

$$C = \frac{D}{d} \text{ is the spring index.}$$

The more accurate value of wire diameter can be found from,

$$d = \sqrt[3]{\frac{8\ K\ P\ D}{\pi\ e_{s\varepsilon}\ \sigma_a}}$$

If there are no space limitations, then set

$D = cd$

and select a suitable value for c which will determine the value of k. The wire diameter (d) can then be found from,

$$d = \sqrt{\frac{8\ P\ c\ k}{\pi\ \sigma_a}}$$

The wire diameter can the be standardised.

The number of active coils can be determined from the equation,

$$i = \frac{y\ d^4\ G}{8\ P\ D^3}$$

If y is not known it can be determined from the spring scale.

If i becomes too small, the spring is soft. According to equation for number of coils, the pitch diameter (D), should be decreased. This will also decrease wire diameter (d) and the calculations must be repeated. If i is too large, D must be increased and the calculations repeated.

The minimum free length ($l_o$) can be determined from,

$$l_o \geq (i + 2)\, d + y + a$$

where, a is added to prevent the spring from being compressed solid.

A helical compression spring having a great length in proportion to its pitch diameter may buckle. The critical axial load ($P_{cr}$) that can cause buckling can be found from,

$$P_{cr} = P_o\, k_1\, l_o$$

where,

$k_1$ is the factor depending on the ratio $l_o/D$ and can be read from Fig. 1.5 [4].

1.5 <u>Present Work</u>:

Computer aided design (CAD) is a technique in which man and machine together form a team to solve problems. The best characteristics of each are used and the results obtained are better than that would be obtained by either man or machine alone. There exists a clear division between the function of man and computer in CAD [5].

The computer has the following three main functions:

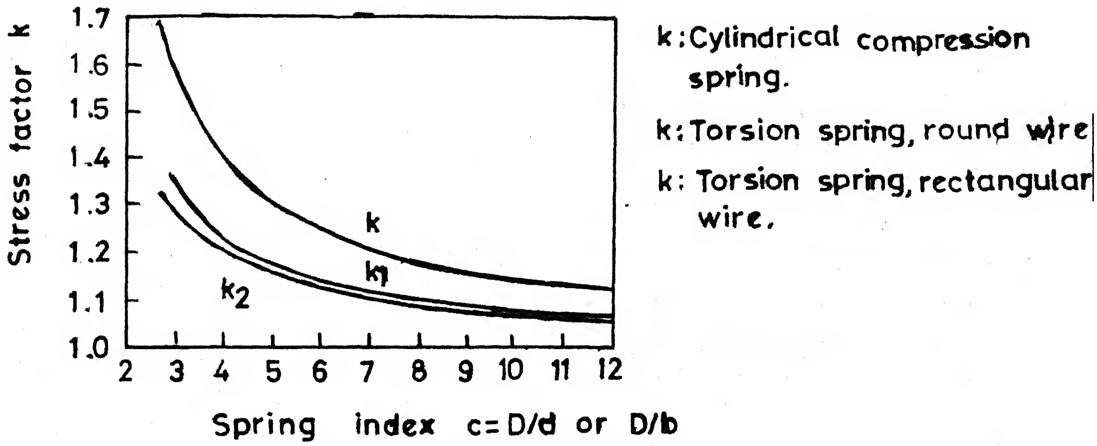



Fig.1.4 Stress factors for helical springs.

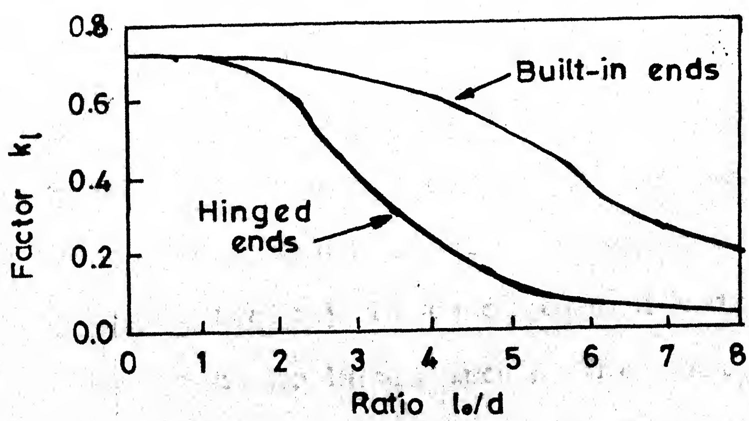



Fig.1.5 Buckling factor for helical springs.

a) To serve as an extension to the memory of the designer.

b) To enhance the analytical and logical power of the designer.

c) To relieve the designer from routine repetitious tasks.

The designer has the following functions:

a) Control of the design process in information distribution.

b) Application of creativity, ingenuity, and experience.

c) Organisation of design information [6].

The design of an engineering system can be broadly classified into five categories which can be wholly or partially turned over to the computer. In the present work the design of leaf springs is discussed in detail.

The specification stage consists of formulating requirements for the design with respect to the environment in which the final product is to be manufactured and used. The design strategy is to develop a principle on which the design is to be based upon. The design principles for leaf spring are discussed in Chapter 2.

The design solution involves resolving a final design by iterative design, direct design or design analysis. The final algorithm for the design is discussed in Chapter 3. The checking stage tests the validity of the final design and ensures that assumptions taken in each stage are still valid at its completion. The application stage consists of manufacturing the design or of providing sufficient information to enable the design to be carried out without any further design effort. The program has been developed for design of leaf springs based on the algorithm given in Chapter 3. The results of few problems have been given in Chapter 4.

# CHAPTER - 2

## DESIGN OF LEAF SPRINGS

### 2.1 General Characteristics:

The leaf spring serves to absorb and store energy and then release it. During this storage of energy in the spring, the deflection must not exceed a certain maximum value to avoid premature failure. This will limit the maximum energy that can be stored.

Leaf springs are used in automotive applications, since they can function as structural members also. When a leaf spring is used as an attaching linkage, it will guide the supported members in a certain geometrical path. If no other guiding members are used, the desired geometry must be obtained by properly placing the supporting parts on the structure which carries the spring. If other guiding members are used, their geometry must fit that of the spring or forces may be set up that will cause failure. The geometry of spring action are explained in sections 2.3, 2.4 and 2.5[7].

## 2.2 Leaf Springs for Vehicle Suspension:

The specifications for a vehicle suspension are given in Appendix I. As explained in Section 2.1, the leaf spring serves to store energy. This energy is represented by the area under the load-deflection diagram as shown in Fig. 2.1. The weight of a spring for a given maximum stress is determined by the energy which is to be stored. The effect of changes in rate and clearance on weight can be seen from Fig. 2.1.

In the case of the stiff spring, energy and weight can be decreased by making the spring softer. In the case of the flexible spring, energy and weight can be decreased by making the spring stiffer. The dividing point between these two cases is,

Static deflection = Ride clearance

It can be seen from Fig. 2.1 that the change in clearance will affect the stored energy of the stiff spring much more than that of the soft spring. This change in stored energy will affect the weight of the stiff spring much more than that of the soft spring.

As can be seen from Fig. 2.1, a flexible spring will have a large static and total deflection. A large static deflection of the suspension will give a " soft ride". The static deflection to be used depends upon

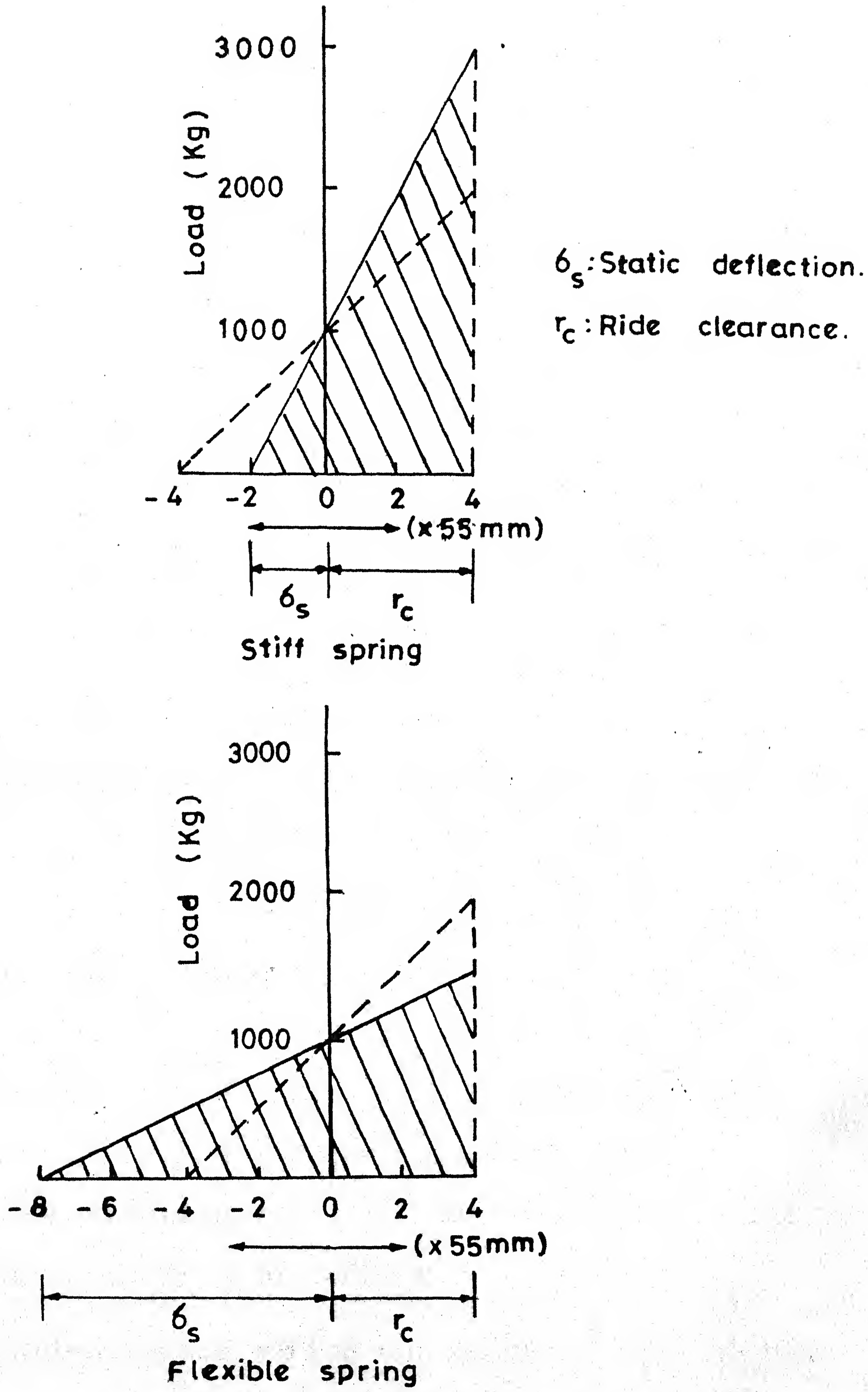



Fig. 2.1 Load-deflection curve of stiff and flexible spring for the same load and clearance.

the available ride clearance. It also depends upon the size of the vehicle because of consideration of stability in braking, accelerating, etc. Table 2.1 shows typical static deflections and ride clearances.

## 2.3 Deflection Theory:

A uniform strength leaf spring is a spring with leaves of constant cross-section properly stepped to approach the condition of uniform strength. When a uniform strength leaf spring is deflected, it will assume the shape of a circular arc at all loads, provided it has a circular arc or is flat at no load or at any given load (initial). Leaf springs approximate this condition closely enough so that the circular shape can be used to calculate their geometric properties.

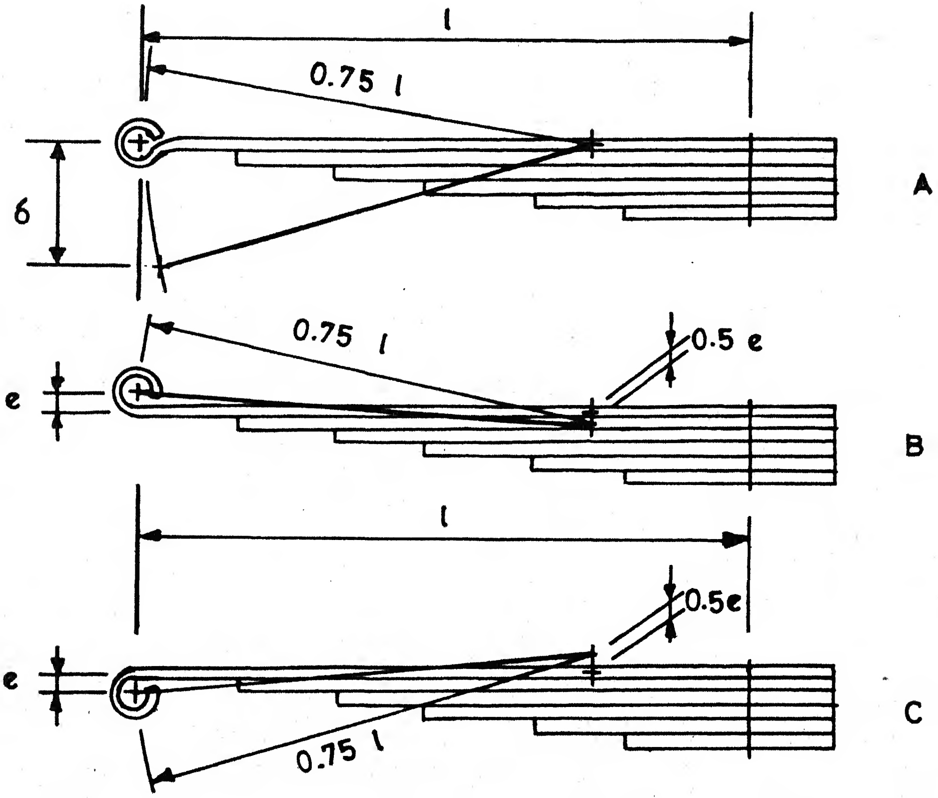
In a cantilever spring, as shown in Fig. 2.2, the center of the Berlin eye moves in a path with radius $0.75\ l$ central to the main leaf. If the eye center is offset by distance " e ", the center of arc will be offset by distance $0.5\ e$ in the opposite direction. Making this construction gives the change of arc height with an accuracy of 1% upto deflections, $\delta = 0.6\ l$.

A semi-elliptic spring can be considered as two cantilever springs and the geometry of the spring action can be determined by considering the spring as a three-

TABLE 2.1

Typical static deflections and side clearances

| | Static deflection mm | Ride clearance mm |
|---|---|---|
| Passenger automobiles at design load | 100 - 300 | 75 - 125 |
| Motor coaches at maximum load | 100 - 200 | 50 - 125 |
| Trucks for | | |
| Highway operation | 75 - 200 | 75 - 125 |
| Off the road operation | 25 - 175 | 50 - 125 |




A: Berlin eye

B: Upturned eye

C: Down-turned eye

Fig.2.2 Equivalent linkage of cantilever spring.

link mechanism as shown in Fig. 2.3. The three-link equivalent layouts determine the path of the axle and the axle control. Axle control is defined as the seat angle (defined in Appendix I) change in degrees per 25 mm of deflection. The correction for shackle effects can also be determined. There are two methods for determining the spring geometry - the center link extension method and the two point deflection method.

## 2.4 Center Link Extension Method:

This construction is based on the principle that every extension of the center link for any position of the linkage will intersect at a common point. The three link equivalent layout is made by starting from the position where the main leaf is flat. The procedure for this method is given in Appendix II

## 2.5 Two Point Deflection Method:

This method has the advantage that all the layout work can be done within the overall length of the spring. When the unsymmetry factor is small, this is the only procedure by which construction can be made. The principle of this method is that the two cantilever deflections must correspond to a given deflection at the center of the spring seat. The procedure for this method is given in Appendix III.

## 2.6 Rate, Load and Stress in First Approximation:

A leaf spring can be considered as a beam made up of leaves of uniform thicknesses and the stress is the same throughout the length of the beam. The formulae given in Table 2.2 are based on the beams of uniform strength (refer Appendix IV) and the following points can be seen.

a) Stress is proportional to thickness multiplied by change of curvature.

b) Change of curvature is proportional to the change of bending moment divided by the moment of inertia.

The stress from strain formula indicates that for the same change in curvature, the stress will vary directly with the leaf thickness. From the stress from deflection formula it can be seen that the stress will vary directly with the leaf thickness and inversely with the square of the effective spring length. The stress from load formula is the beam formula for stress, where for a given load, the stress will vary directly with effective length and inversely with the square of the leaf thickness. The stress from deflection formula shows that for a given stress and deflection, the leaf thickness varies inversely as the square of the effective

STRESS AND RATE FORMULAE FOR LEAF SPRINGS

| | Symmetrical semi-elliptic | Unsymmetrical semi-elliptic | Uniform strength cantilever | Uniform section cantilever |
|---|---|---|---|---|
| Deflection from geometry | $\delta = \frac{L^2}{8}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\delta = \frac{ab}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$<br>$\delta = \frac{yL^2}{2(y+1)^2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\delta = \frac{L^2}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\delta = \frac{L^2}{3}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ |
| Stress from strain | $\sigma = \frac{Et}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\sigma = \frac{Et}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\sigma = \frac{Et}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ | $\sigma = \frac{Et}{2}\left(\frac{1}{R} - \frac{1}{R_o}\right)$ |
| Stress from deflection | $\sigma = \frac{4Et}{L^2}.\delta.SF$ | $\sigma = \frac{Et}{ab}.\delta.SF$<br>$\sigma = \frac{Et}{L^2}\,\frac{(y+1)^2}{y}.\delta.SF$ | $\sigma = \frac{Et}{L^2}.\delta.\ SF$ | $\sigma = \frac{1.5Et}{L^2}.\delta$ |
| Stress from load | $\sigma = \frac{3}{2}\,\frac{LtP}{WNt^3}$ | $\sigma = \frac{6abt}{WNt^3L}.P$<br>$\sigma = \frac{6\ Lt}{WNt^3}.\frac{y}{(y+1)^2}.P$ | $\sigma = \frac{6\ Lt}{WNt^3}.P$ | $\sigma = \frac{6\ Lt}{Wt^3}.P$ |
| Load rate | $K = \frac{P}{\delta} = \frac{8E}{3}.\frac{WNt^3}{L^3}.SF$ | $K = \frac{E}{6}.\frac{WNt^3L}{a^2b^2}.SF$<br>$K = \frac{E}{6}.\frac{WNt^3}{L^3}.\frac{(y+1)^4}{y^2}.SF$ | $K = \frac{E}{6}.\frac{WNt^3}{L^3}.SF$ | $K = \frac{E}{4}\,\frac{Wt^3}{t^3}$ |
| Strain energy stored per $mm^2$ | $\sigma^2/6E$ | $\sigma^2/6E$ | $\sigma^2/6E$ | $\sigma^2/18E$ |
| Length ratio | $y = 1$ | $y = b/a$ | | |

Note: Unloaded springs have a radius of curvature $R_o$ which is considered negative.

spring length. Since thin leaves will not give sufficient strength for spring eyes, the formula emphasizes the need of long springs.

## 2.7 Stiffenning Factor:

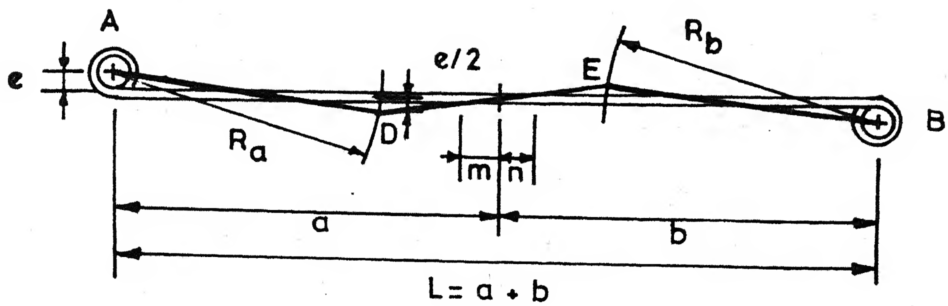
The actual leaf springs are not truly beams of uniform strength. The length of the leaves, leaf ends, and the center clamp affect the uniform strength characteristic.

Sometimes two or more full length leaves are used. The shorter leaves may be longer than they would be for uniform strength beam. These are done in order to reduce the main leaf stress in the area of the eyes. It can be seen from Fig. 2.4 that the ends of the leaves exceed the outline of the triangular beam. This will make the spring stiffer. Since the leaf springs are used with clamps, the leaf lengths have to be designed for clamped springs. The effect of clamp can be taken care of by using the active length instead of full length. The influence of these factors can be taken into account by a factor called as the stiffenning factor. It can be calculated from,

$$SF = 1 + \frac{n}{2N} \qquad (2.1)$$

where,

n - is the number of full length leaves including main leaf.




$R_a = 0.75(a-m)$

$R_b = 0.75(b-n)$

A-D-E-B: 3- Link layout.

Fig.2.3 Typical 3-link layout for upturned - down-turned eye.

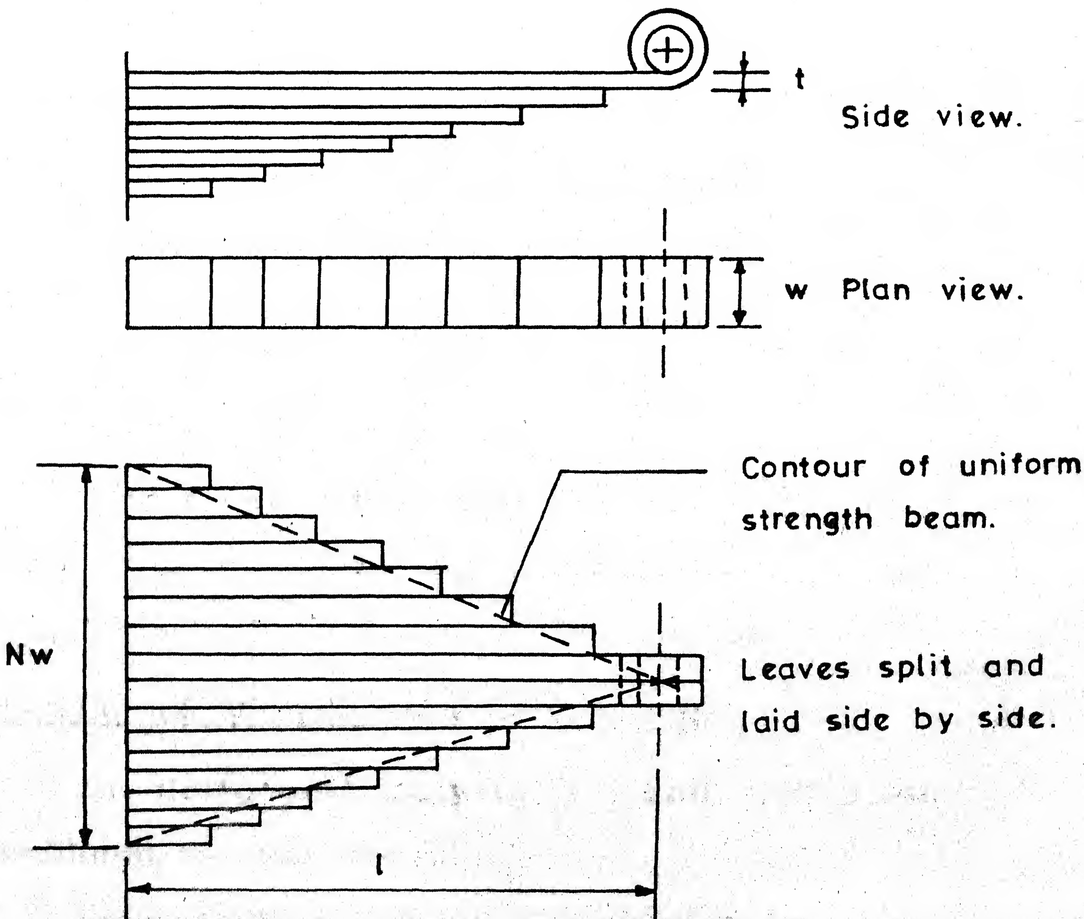



Fig.2.4 Description of a multi-leaf spring.

N - is the total number of leaves in the spring.

The maximum value of stiffenning factor (SF) is 1.5, when there is only one leaf which is full length or when all leaves are full length. For a uniform strength cantilever SF is 1.0, if the design provides for a truly uniform strength beam.

2.8 Weight of Active Spring:

The approximate, necessary weight of an active spring can be derived from the strain energy stored due to work done.

$$\frac{1}{2} P\delta = \frac{\sigma^2}{6E} \cdot V$$

Substituting,

$$V = \frac{W}{\rho} \quad \text{and}$$

$$P = K\delta$$

We get,

$$W = \frac{3 E \rho \delta^2 K}{\sigma^2} \qquad (2.2)$$

This formula shows that regardless of length and width, the spring will require certain weight of material to do a given job at a given stress.

2.9 Design and Analysis of a Leaf Spring:

The design and analysis of a leaf spring can be considered in four stages.

a) Approximate design as in section 2.10.

b) Determination of leaf lengths and individual leaf radii from a desired stress distribution as shown in section 2.11.

c) Analysis of stresses and stress ranges at various points in the spring by common curvature or point pressure as shown in section 2.12.

d) To check the soundness of assumptions involved in (c), and analysis by a combination of common curvature and point pressure or by strain gage measurements on the spring.

## 2.10 Preliminary Design:

In this section an approximate estimate of rate (K), leaf thickness (t) and number of leaves is made. A stiffenning factor of 1.08 is initially used. Using the formulae of Table 2.2, the thickness of leaves and the number of leaves can be determined. For unsymmetrical springs, where length ratio does not exceed 1.30, the symmetrical formulae can be used. The procedure for preliminary calculation is as follows.

A) Given design load (P), side clearance ($r_c$) length (L), allowable stress ($\sigma_{max}$) and rate (K). If rate is not given it can be calculated from the design load and static deflection ($\delta_s$).

$$K = \frac{P}{\delta_s}$$

Maximum deflection,

$$\delta_{max} = \delta_s + r_c$$

To calculate rates for half-elliptic cantilever springs, where one cantilever end is made of more leaves than the other, or where its leaf lengths are extended beyond the uniform strength requirements, it can be considered as made up of two cantilever springs. The rates of both cantilevers are calculated separately and combined by the use of the following formulae.

$$K = \frac{(y + 1)^2}{z + y^2} \quad K_a \quad \text{or}$$

$$K = \frac{L^2}{\frac{a^2}{K_b} + \frac{b^2}{K_a}}$$

B) Find the permissible leaf thickness from the stress from deflection formula of Table 2.2

$$t_{max} = \frac{L^2}{4E} \; \frac{\sigma_{max}}{\delta_{max}} \; \frac{1}{SF}$$

Thinner leaves will give a lower stress, and thicker leaves a higher stress. The standard thicknesses can be selected from Table 2.3.

C) Find the number of leaves and the leaf width which will give the required rate from the formula for load rate from Table 2.2

TABLE 2.3

Standard thickness of leaf springs from IS:1135-1973

| | | | | | |
|---|---|---|---|---|---|
| Nominal thicknesses in mm are: | 3.2 | 4.5 | 5.0 | 6.0 | 6.5 |
| | 7.0 | 7.5 | 8.0 | 9.0 | 10.0 |
| | 11.0 | 12.0 | 13.0 | 14.0 | 16.0 |
| Nominal width in mm are: | 32 | 40* | 45 | 50* | 55 |
| | 60* | 65 | 70* | 75 | 80 |
| | 50* | 100 | 125 | | |

* Preferred widths

$$wN = \frac{3}{8E} \cdot \frac{KL^3}{t^3} \cdot SF$$

The width (w) if not given can be chosen so that the minimum number of leaves is six.

As a check on these calculations the active weight of the spring can be calculated from the actual dimensions and from Equation 2.2 and compared. The weight of spring from actual dimensions,

$$W = \frac{\rho \cdot Nt \cdot w \cdot (L/2)}{SF}$$

## 2.11 Stress Distribution:

After the preliminary design is over, specifications such as standard thicknesses, lengths and free radius of each leaf have to be given. For a desired stress distribution, the thicknesses, leaf lengths and individual leaf radii can be determined.

### 2.11.1 Leaf Thickness

The main leaf is generally made one gage thicker and several short leaves one gage thinner than the intermediate leaves. This is done

a) to give the main leaf more strength to resist eye forces,

b) to compensate for the difference in leaf radii (assembled and unassembled radii),

c) because desired rates can be approached more closely by a combination of standard gages.

2.11.2 Leaf Radii:

The free radii of leaves are different and the curvature becomes more and more negative from main leaf towards the shortest leaf. When assembled the leaves are under some stresses called as assembly stresses. In main leaf the assembly stress is opposed to load stress and in short leaves it is additive to load stress. This reduces the main leaf stress.

2.11.3 Stepping of Leaves:

The lengths of the leaves of a spring with thicknesses and the individual leaf radii determine the distribution of stresses along each leaf. The shape of leaf under load and its rate can also be controlled.

The leaves of a spring bear on each other mainly on a relatively small area near the leaf tip. The center of pressure of this area is called as the " bedding point". The distance from one bedding point to that of the next shorter leaf is called as the step or overhang. The active length of the spring can be written as,

$$\frac{L}{2} = \sum_{i=1}^{N} L_i' \qquad (2.4)$$

where,

$L_i'$ is the overhang of the $i^{th}$ leaf

L active length of spring (total)

To obtain uniform stress along the length of each leaf at a given load, it is necessary that each leaf be subjected to only pure coupler applied to its ends i.e,

$$\sigma_i = \frac{6 \, P \, L_i'}{w \, t_i^2}$$

Since load and width are constants,

$$L_i' \; \alpha \; \sigma_i \cdot t_i^2 \tag{2.5}$$

Taking summation,

$$\sum_{i=1}^{N} L_i' \; \alpha \; \sum_{i=1}^{N} \sigma_i \cdot t_i^2 \quad \text{or}$$

$$\frac{L}{2} \; \alpha \; \sum_{K=1}^{N} \sigma_K \cdot t_K^2 \tag{2.6}$$

From Equations 2.5 and 2.6, we get,

$$\frac{L_i'}{L/2} = \frac{\sigma_i \cdot t_i^2}{\sum\limits_{K=1}^{N} \sigma_K \cdot t_K^2}$$

Overhang or step is given by,

$$L_i' = \frac{L}{2} \; \frac{\sigma_i \cdot t_i^2}{\sum\limits_{K=1}^{N} \sigma_K \cdot t_K^2} \tag{2.7}$$

2.11.4 Assembly Stresses:

The free radii of leaves are different and when they are assembled, the leaves are under some stresses. These stresses are called assembly stresses. Let $q_i$ be the curvatures of different leaves when unassembled. In assembly (unloaded) a common curvature $q_o$ is established. The common curvature $q_o$ can be calculated from the condition that the internal bending moments of all the leaves must cancel when the spring is assembled but not loaded. This is represented in the equation form as,

$$\sum_{i=1}^{N} (\sigma_{ash})_i \; t_i^2 = 0 \qquad (2.8)$$

The assembly stresses are chosen arbitrarily except that they must become increasingly larger in going from the long to short leaves and the sum given by Equation 2.8 must equate to zero.

2.12 Analysis of Stresses and Stress Ranges:

The stresses and stress ranges at various points in a spring can be analysed according to two assumptions about the action of leaf springs. They are " Point pressure" and " Common curvature".

Point pressure means that the leaves touch each other only at the bearing points and at the center

clamp. Tip loads of successive leaves are calculated from the condition that two contacting leaves must have a common load and deflection. After the tip loads have been calculated each leaf can be analysed as a simple beam. This assumption is justified when leaves are free to take the shape which corresponds to the load distribution. This is not possible unless spacers are provided between them.

Common curvature means that all the leaves of a spring touch their neighbours along their length. This assumption is justified at all points where a leaf is surrounded by other leaves. Calculations based on this are relatively simple and are given below.

A bending moment (M) at any cross-section will produce a change of curvature (q), which is given by,

$$q = \frac{M}{E \Sigma I_n} \qquad (2.9)$$

where,

$I_n$ - is the moment of inertia at that section.

The change of curvature (q) is given by,

$$q = \frac{1}{R_{loaded}} - \frac{1}{R_{free}}$$

Sign Convention for Curvature:

Curvature is zero when leaf is flat and positive under heavier loads.

The load stress ($\sigma_p$) in any leaf is given by,

$$\sigma_p = q \, E_y$$

where,

y is the distance from the neutral axis to the remotest fiber in tension. For rectangular section leaves with thickness (t),

$y = \frac{t}{2}$ and substituting for q from Equation 2.9

we get, $\sigma_p = \dfrac{6 \, M \, t}{w \, \Sigma \, t^3}$

If all the leaves are of same thickness,

$$\sigma_p = \frac{6 \, M}{w \, \Sigma \, t^2} \qquad (2.10)$$

The total stress in any leaf is the sum of the assembly stress and the load stress.

$$\sigma = \sigma_{asb} + \sigma_p \qquad (2.11)$$

The load stress would equal the total stress if the leaves are fitted " dead " in a spring, i.e. all the leaves have the same unassembled curvature in which case assembly stresses of all leaves are zero.

## 2.13 Sample Calculation:

A sample example is solved for an unsymmetrical semi-elliptic leaf spring. The results are tabulated in a series a Tables. The data for the problem is as follows.

| | | |
|---|---|---|
| Rate | $K$ | $= 1.79$ kgf/mm |
| Design load | $P$ | $= 340$ kgf |
| Opening at design load (- ve) | | $= - 190$ mm |
| Eye-diameter | | |
| front upturned | $e_a$ | $= 38.0$ mm |
| rear upturned | $e_b$ | $= 38.0$ mm |
| Clearance | $r_c$ | $= 115.0$ mm |
| Length | $L$ | $= 1372.0$ mm |
| Length | | |
| front cantilever | $a$ | $= 560.0$ mm |
| rear cantilever | $b$ | $= 812.0$ mm |
| Width | $w$ | $= 50.0$ mm |
| Maximum stress | $\sigma_{max}$ | $= 98.28$ kgf/mm$^2$ |
| Length of seat clamp | | $= 100.0$ mm |
| Distance between edges of clamp bolts | | $= 75.0$ mm |
| Young's modulus of elasticity | $E$ | $= 20358$ kgf/mm$^2$ |

The cantilever ratio $y = b/a = 1.45$ exceeds 1.30 and therefore the unsymmetrical formula of Table 2.2 must be used.

a) Thickness of leaves and number of leaves.

Total deflection = static deflection + clearance

$$\text{Static deflection} = \frac{\text{design load}}{\text{Rate}}$$

$$\delta_s = P/K$$

$$\therefore \quad \delta_{max} = \frac{P}{K} + r_c = 304.6 \text{ mm}$$

Maximum load is given by,

$$P_{max} = \delta_{max} \cdot K$$

$$P_{max} = 545 \text{ kgf.}$$

Leaf thickness is given by,

$$t_{max} = \frac{L^2}{E} \cdot \frac{y}{(y+1)^2} \cdot \frac{\sigma_{max}}{\delta_{max}} \cdot \frac{1}{SF}$$

$$t_{max} = 6.67 \text{ mm}$$

The number of leaves is given by,

$$N = \frac{6K}{E} \cdot \frac{L^3}{wt^3} \cdot \frac{y^2}{(y+1)^4} \cdot \frac{1}{SF}$$

$$N = 4.95 \text{ leaves.}$$

All the leaves i.e. 4.95 leaves are of 6.67 mm thickness.

b) Grading with standard gages

The main leaf is made one gage thicker and by trial and error a combination of leaves is found out such that $X \geq X_1$. $(X = Nt^3 = 1448.87)$

The combination of leaves is given in Table 2.4.

A stiffenning factor of 1.08 was used in the calculation. A check on this can be made.

$$SF = 1 + \frac{n}{2N}$$

$$= 1.083$$

Since the estimated value and the value used in calculation are close , no further correction is necessary.

c) Stress distribution between leaves

For calculation of stresses, the inactive length of the spring due to seat clamp has to be subtracted.

Therefore,

Active length for front cantilever,

$$l_a = 522.5 \text{ mm}$$

Active length for rear cantilever,

$$l_b = 774.5 \text{ mm}$$

Front cantilever stress,

$$\sigma = \frac{6\ l_a\ t}{w\ \Sigma\ t^3}\ ,\ P_a\ ,\quad \text{where}\quad P_a = \frac{P.b}{L}$$

$$\sigma = 2.41 \times 10^{-2}\ P,t$$

Rear cantilever stress,

$$\sigma = \frac{6\ l_b\ t}{w\ \Sigma\ t^3} \cdot P_b\ ,\quad \text{where}\quad P_b = \frac{P.a}{L}$$

$$\sigma = 2.46 \times 10^{-2}\ P,t$$

The stress at design and maximum load are given in Table 2.5.

By using different leaf radii, assembly stresses can be added to or deducted from load stress to obtain a desired stress distribution. The

TABLE 2.4

SAMPLE EXAMPLE : GRADING WITH STANDARD GAGES

| Leaf No. | Gage t, mm | $t^3$ | $\Sigma t^3 N$ |
|---|---|---|---|
| 1 | 7.0 | 343.00 | 343.00 |
| 2,3 | 6.5 | 274.63 | 549.25 |
| 4,5,6 | 6.0 | 216.00 | 648.00 |

$X_1 = 1540.25$

TABLE 2.5

SAMPLE EXAMPLE : STRESS AT DESIGN AND MAXIMUM LOAD

| Leaf No. | t,mm | Front cantilever | | Rear cantilever | |
|---|---|---|---|---|---|
| | | des kgf/mm$^2$ | max, kgf/mm$^2$ | des, kgf/mm$^2$ | max, kgf/mm$^2$ |
| 1 | 7.0 | 57.36 | 91.94 | 58.55 | 93.85 |
| 2,3 | 6.5 | 53.26 | 85.37 | 54.37 | 87.15 |
| 4,5,6 | 6.0 | 49.16 | 78.81 | 50.18 | 80.44 |

assembly stresses are chosen arbitrarily except that they must be increasingly larger from long to short leaves, and the sum $\Sigma \sigma_{asb} \cdot t^2$ must equate to zero. They are tabulated in Table 2.6. The total stress in a leaf will be the addition of load stress and assembly stress in that leaf. These are given in Table 2.7.

d) Stepping of leaves.

The overhangs are calculated such that the stresses are uniform along each leaf.

$$\text{Overhang} = \text{Cantilever length} \; \frac{\sigma t^2 \text{ of the leaf}}{\Sigma \sigma t^2 \text{ of all the leaves}}$$

The overhangs are given in Table 2.8

The leaf lengths are obtained by adding the successive overhangs. For the shortest leaf 37.5 mm per end must be added for the inactive center portion (75 mm total) and at least 25 mm for distance from bedding point to leaf tip. They are given in Table 2.9.

## 2.14 Variable Rate Leaf Springs:

Variable rate springs are used on vehicles which operate with large variations in load such as trucks and buses. Variable rate springs are required to provide desirable ride and handling characteristics.

TABLE 2.6

SAMPLE EXAMPLE : DISTRIBUTION OF ASSEMBLY STRESSES

| Leaf No. | t,mm | $t^2$ | asb | asb $t^2$ |
|---|---|---|---|---|
| 1 | 7.0 | 49.00 | -19.66 | -963.34 |
| 2 | 6.5 | 42.25 | - 0.83 | - 35.07 |
| 3 | 6.5 | 42.25 | 4.28 | 180.83 |
| 4 | 6.0 | 36.00 | 6.61 | 237.96 |
| 5 | 6.0 | 36.00 | 7.48 | 296.28 |
| 6 | 6.0 | 36.00 | 7.87 | 283.32 |

TABLE 2.7

SAMPLE EXAMPLE : TOTAL STRESS IN EACH LEAF

| Leaf No. | t, mm | Front cantilever | | Real cantilever | |
|---|---|---|---|---|---|
| | | des, $kgf/mm^2$ | max, $kgf/mm^2$ | des, $kgf/mm^2$ | max, $kgf/mm^2$ |
| 1 | 7.0 | 37.70 | 72.28 | 38.89 | 74.19 |
| 2 | 6.5 | 52.43 | 84.54 | 53.54 | 86.32 |
| 3 | 6.5 | 57.54 | 89.65 | 58.65 | 91.43 |
| 4 | 6.0 | 55.77 | 85.42 | 56.79 | 87.05 |
| 5 | 6.0 | 56.64 | 86.29 | 57.66 | 87.92 |
| 6 | 6.0 | 57.03 | 86.68 | 58.05 | 88.31 |

## TABLE 2.8

Sample example : overhang of each leaf

| Leaf No. | t mm | $t^2$ $mm^2$ | Front cantilever: des $kgf/mm^2$ | Front cantilever: $t^2$ | Rear Cantilever: des $kgf/mm^2$ | Rear Cantilever: $t^2$ | Overhangs: Front mm | Overhangs: Rear mm |
|---|---|---|---|---|---|---|---|---|
| 1 | 7.0 | 49.00 | 37.70 | 1847.30 | 38.89 | 1905.61 | 76.65 | 114.81 |
| 2 | 6.5 | 42.25 | 52.43 | 2215.17 | 53.54 | 2262.07 | 91.91 | 136.28 |
| 3 | 6.5 | 42.25 | 57.54 | 2431.07 | 58.65 | 2477.96 | 100.87 | 149.29 |
| 4 | 6.0 | 36.00 | 55.77 | 2007.72 | 56.79 | 2044.44 | 83.30 | 123.17 |
| 5 | 6.0 | 36.00 | 56.64 | 2039.04 | 57.66 | 2015.76 | 84.60 | 125.05 |
| 6. | 6.0 | 36.00 | 57.03 | 2053.08 | 58.05 | 2089.80 | 85.18 | 125.[illegible] |
| | | | $\Sigma\ t^2$ = | 12593.38 | $\Sigma\ t^2$ = | 12855.64 | 522.51 | 774.51 |

## TABLE 2.9

Sample example : leaf lengths

| Leaf No. | leaf lengths, mm: Front | leaf lengths, mm: Rear |
|---|---|---|
| 1 | 560.01 | 812.01 |
| 2 | 508.36 | 722.20 |
| 3 | 416.45 | 585.92 |
| 4 | 315.58 | 436.63 |
| 5 | 232.28 | 313.46 |
| 6 | 147.68 | 188.40 |

There are several methods to obtain variable rates. The helper spring is one method of obtaining increased rate with deflection and is as shown in Fig. I-5. It is mounted above the main spring and has its own bearing pads. The helper spring does not support any load until contact is made. The change hence becomes abrupt. Another method to obtain variable rate is by means of the multistage spring as shown in Fig. 2.5. This spring has one or more leaves called second stage leaves mounted adjacent to the shortest leaf of the main or first stage of the spring. This spring gradually increases in rate with deflection as the contact between the stages increases. Load and rate for each stage are specified as shown in Fig. 2.5. Shackles may be used to provide some variation in rate. Curved bearing pads or cams which shorten the affective length of a spring as it is deflected will also provide variable rate. This is shown in Fig. 2.6.

Combinations of the above methods may be used to provide a greater change of spring rate.

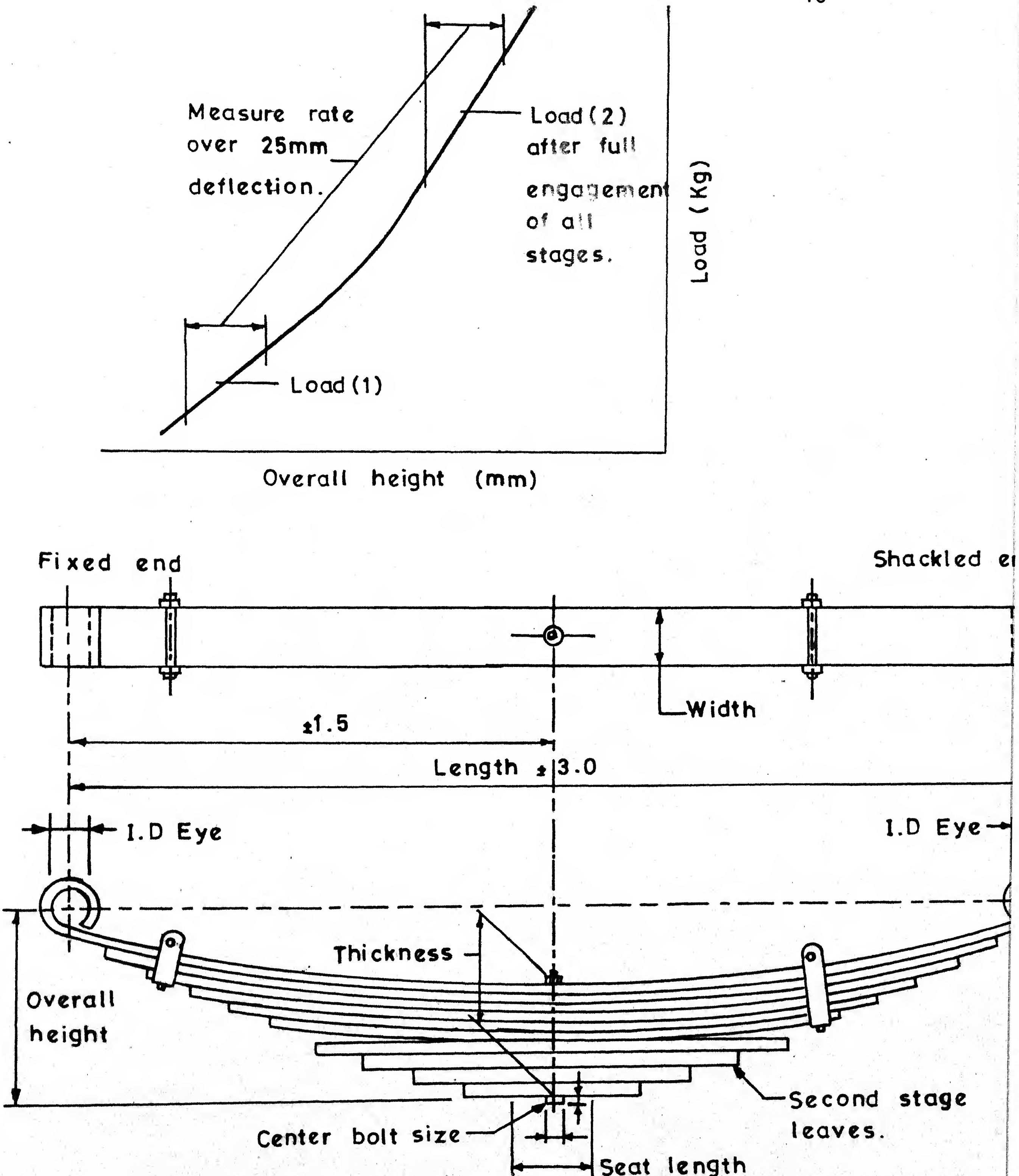



Fig. 2.5 Multi-stage leaf spring.

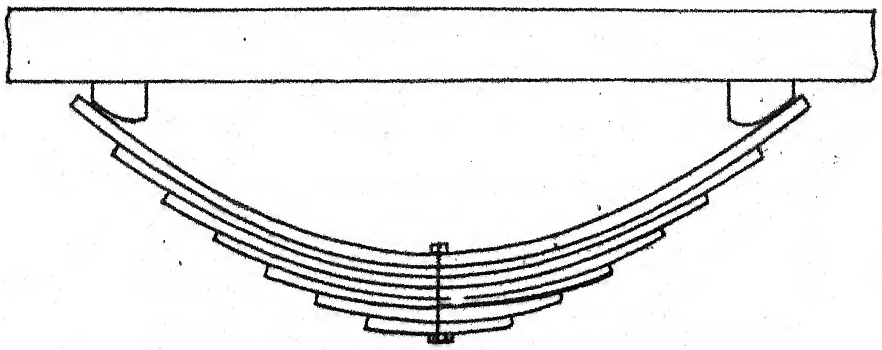


Fig. 2.6 Variable effective length spring.

# CHAPTER - 3

## ALGORITHMS AND FLOW CHARTS

### 3.1 Introduction:

The complex nature of engineering systems and the lengthy iterations and repetitive calculations involved make the task of a designer difficult. The engineering design function can be divided into five categories which can be wholly or partially turned over to the computer. This has been explained in Section 1.5.

The algorithm for leaf spring design and the variable rate leaf spring design have been developed. The user has some control over the intermediate results as well as the final results while the program is in execution mode. In this chapter the algorithms have been explained in detail. The various subroutines and the variables used in the program are also given.

### 3.2 Algorithm for Leaf Spring Design:

The algorithm for leaf spring design is divided into eighteen steps which include the input of data, calculations, interactive features and output of results.

<u>Step 1</u>: Read design load, maximum allowable stress, leaf overhangs, width, Young's modulus of elasticity, lining, endcropping, steel density, assembly camber, error in stress distribution, tolerance in width, length and radius.

<u>Step 2:</u> Read static deflection and ride clearance.
Find stiffness
[Stiffness = design load/static deflection]

<u>Step 3:</u> If you want to change the value of stiffness go to step 2 else continue.
Find maximum deflection and load.

<u>Step 4</u>: Read stiffenning factor.
[It can be read from table if user requires].

<u>Step 5</u>: Find total moment of inertia.
[Load rate formula from Table 2.1].
Find maximum allowable leaf thickness.
[Stress from Load formula from Table 2.1].

<u>Step 6</u>: Call subroutine LFCOMB.
Enter subroutine.
Read number of sets of leaves.
Read leaf thickness and number of leaves in all sets except last one.
Find total number of leaves.
Find approximate weight of spring.
Return.

<u>Step7</u>: Find total moment of inertia - corrected for the number of leaves.

Call subroutine STRESS.

Enter subroutine.

Find stress at design and maximum load.

[Stress from load formula from Table 2.1]

Return.

<u>Step 8</u>: Find corrected spring rate or stiffness.

If you desire to change stiffness go to step 6 and make a change in the leaf combination else continue.

<u>Step 9</u>: Read eye-pin diameter.

Find assembly radius of curvature.

<u>Step 10</u>: Call subroutine ASSTR.

Enter subroutine.

Find assembly stresses in each leaf.

Return.

<u>Step 11</u>: Subroutine LEAFRA.

Enter subroutine.

Find assembled or pack radius of all leaves.

Find unassembled or free radius of all leaves.

Return.

<u>Step 12:</u> Call subroutine OVHANG.

Enter subroutine.

Find leaf overhangs.

Return.

<u>Step 13:</u> Call subroutine LEAFLN.

Enter subroutine.

Find length of each leaf.

Return.

<u>Step 14:</u> Call subroutine ENWT

Enter subroutine.

Find energy per unit weight.

Find approximate weight of spring.

Return.

<u>Step 15:</u> If you are not satisfied with approximate weight of spring go to step 6 else continue.

<u>Step 16:</u> Call subroutine CAMBER.

Enter subroutine.

Find camber of each leaf.

Return.

<u>Step 17:</u> Add corrections to leaf lengths.

Call subroutine SPRWT.

Enter subroutine.

Find actual weight of spring.

Return.

If you are not satisfied with this weight of spring go to step 16, else continue.

Step 13: Call subroutine STRLEF.

Enter subroutine.

Find stresses in each leaf at rated and maximum load.

Return.

Stop.

End.

The flow chart is shown in Fig. 3.1

## 3.3 Algorithm for Variable Rate Spring Using Helper Spring:

The following assumption is made. The variable rate spring is required to operate with a rate (K) under normal conditions and with a rate ($K_1 > K$) under heavy load. The algorithm developed is as follows.

Step 1: Design the spring for rate (K) as given in Section 3.2, using one set of leaf with thickness (t).

Find total thickness of leaf spring (X).

Step 2: Read the desired value of rate ($K_1$).

Step 3: Design the spring for rate ($K_1$) as given in Section 3.2.

Find total thickness of leaf spring ($X_1$).

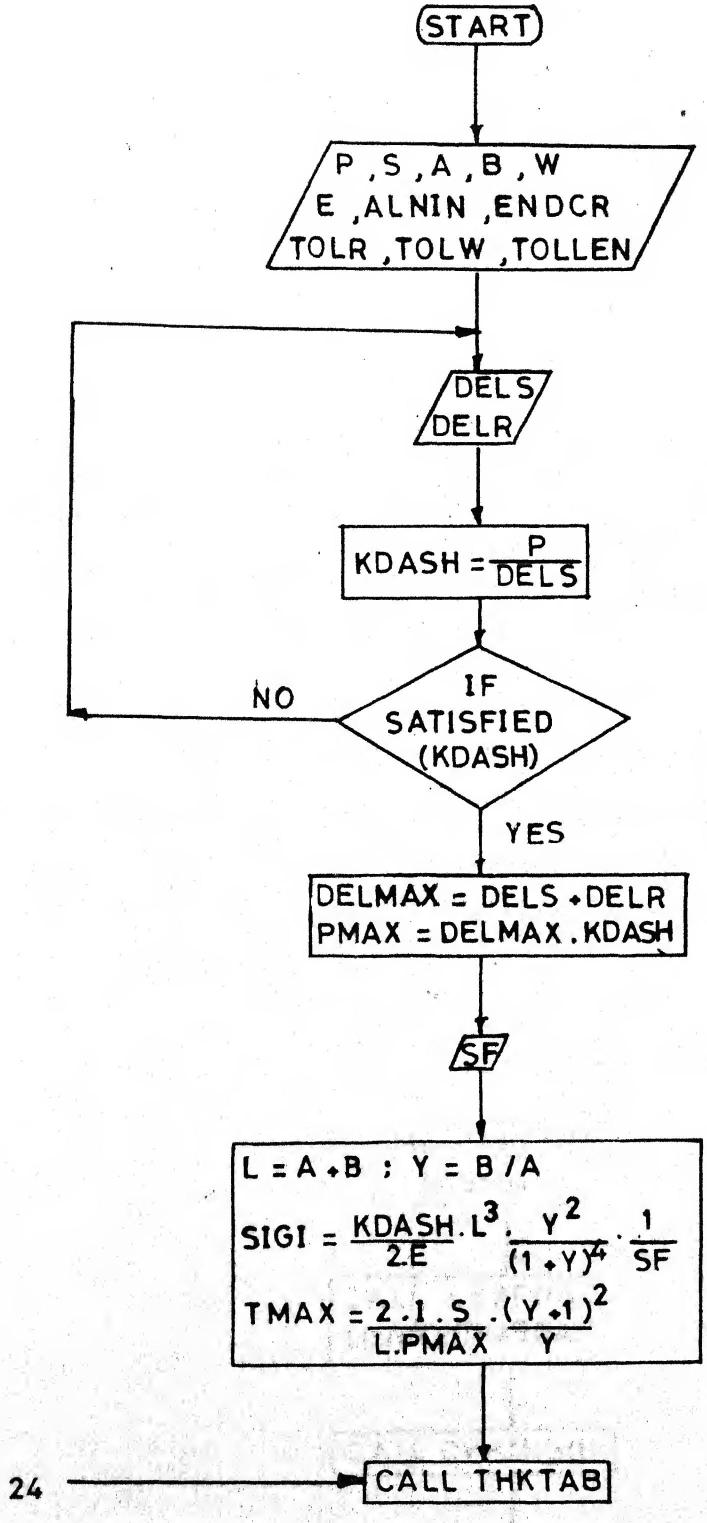



Fig 31 Leaf spring design

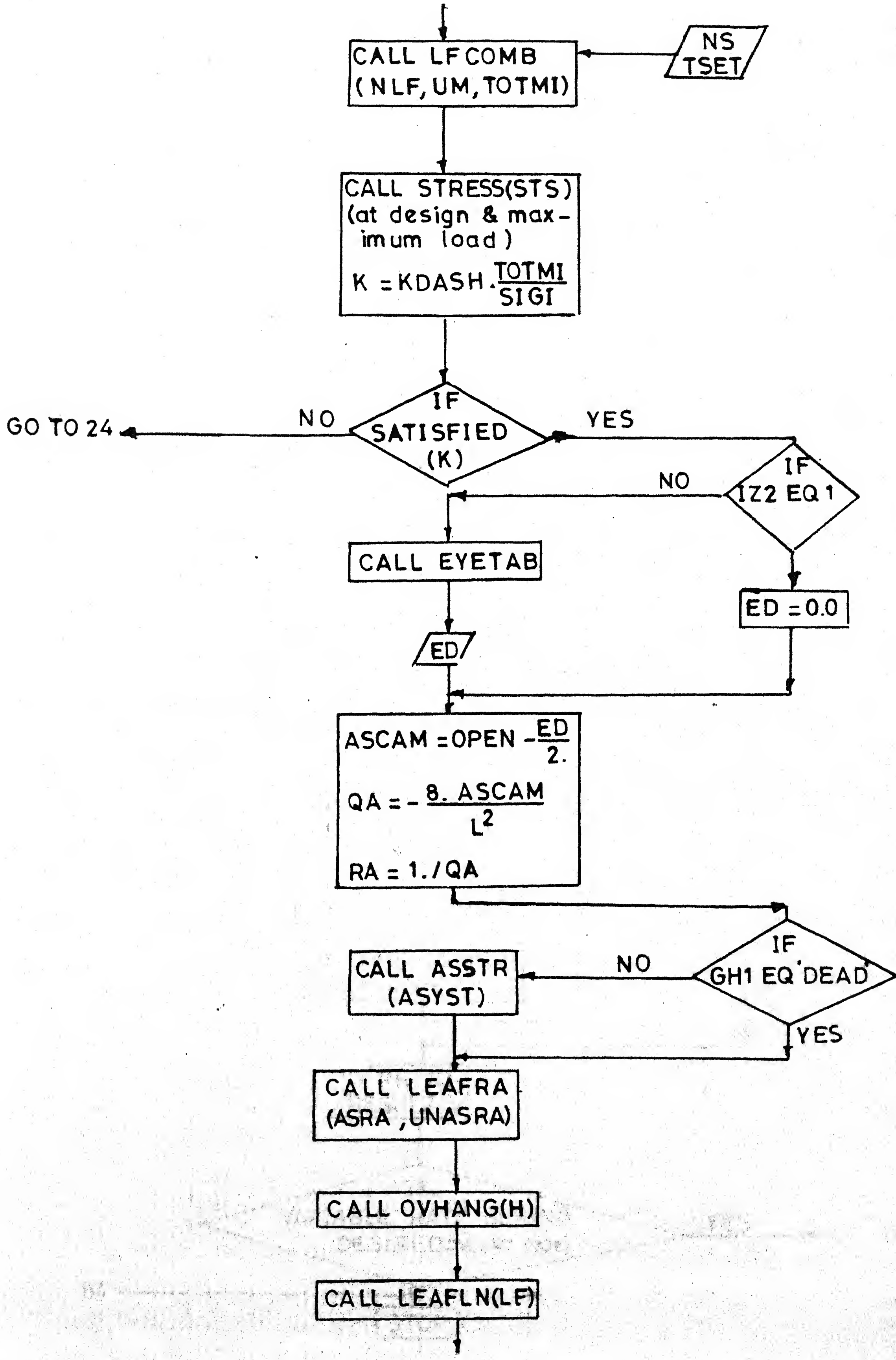



Fig 3.1 Continued

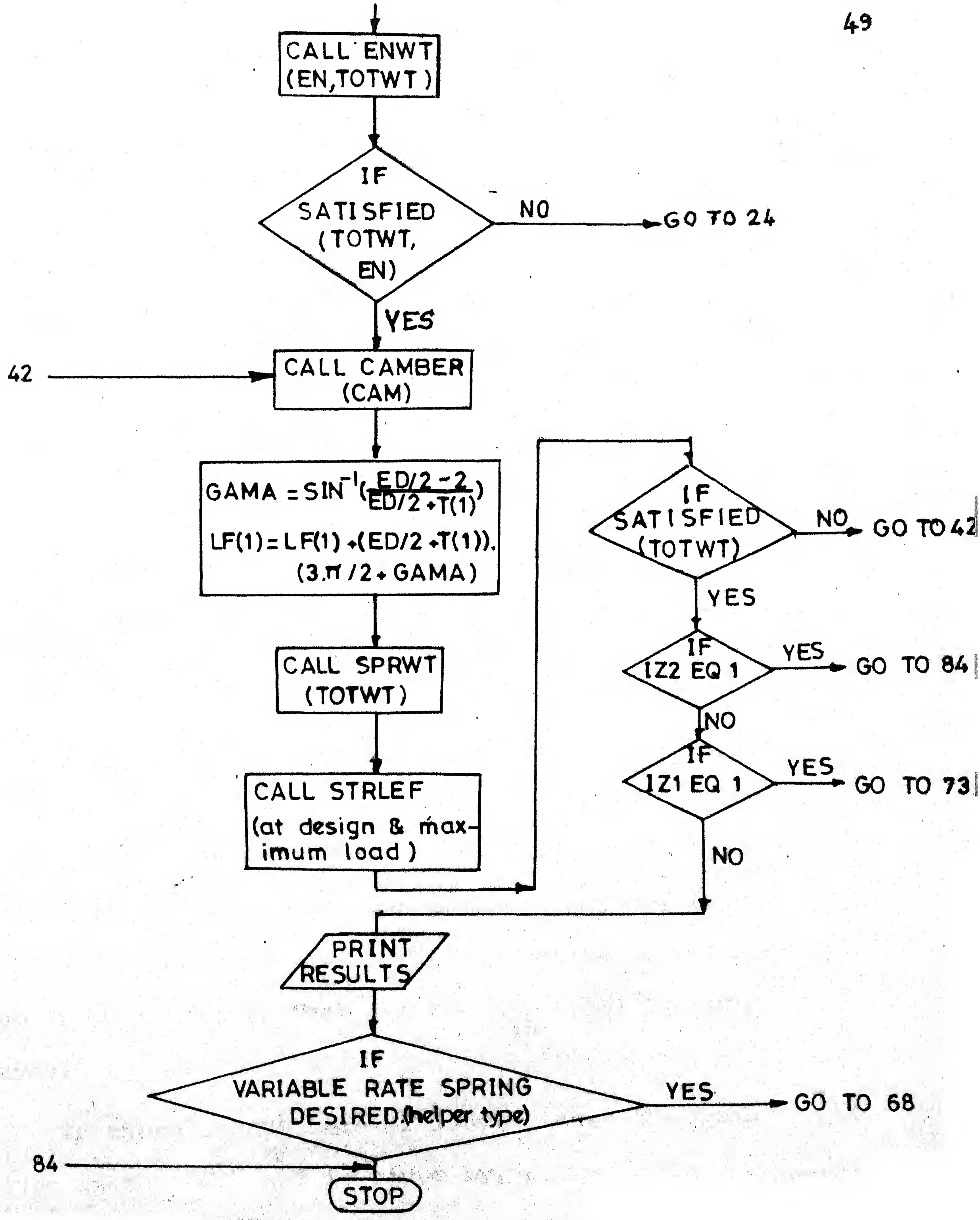



Fig 32 Continued

<u>Step 4:</u> Find total thickness of helper spring ($X_2$)
$[X_2 = X_1 - X]$.
Find stiffness of helper spring, the maximum length of spring (helper), and load coming on the spring.

<u>Step 5:</u> Design the helper spring as given in Section 3.2.
The flow chart is shown in Fig. 3.2.

## 3.4 Subroutines Used in the Program:

There are ten subroutines which are used for calculation of various parameters. Apart from this there are a few subroutines which contain tables for the help of the user. What does each subroutine do? This is explained below.

### 3.4.1 Subroutine LFCOMB

It reads the number of sets of leaves. If only one set is used, it proceeds to calculate the number of leaves and the initial approximate weight of the spring. If more than one set is desired, then read in the thickness and number of leaves in all sets except last one, for which only thickness is read. The subroutine calculates the number of leaves in that set and the total number of leaves.

If there are NS sets of leaves, then the number of leaves in the $NS^{th}$ set is given by,

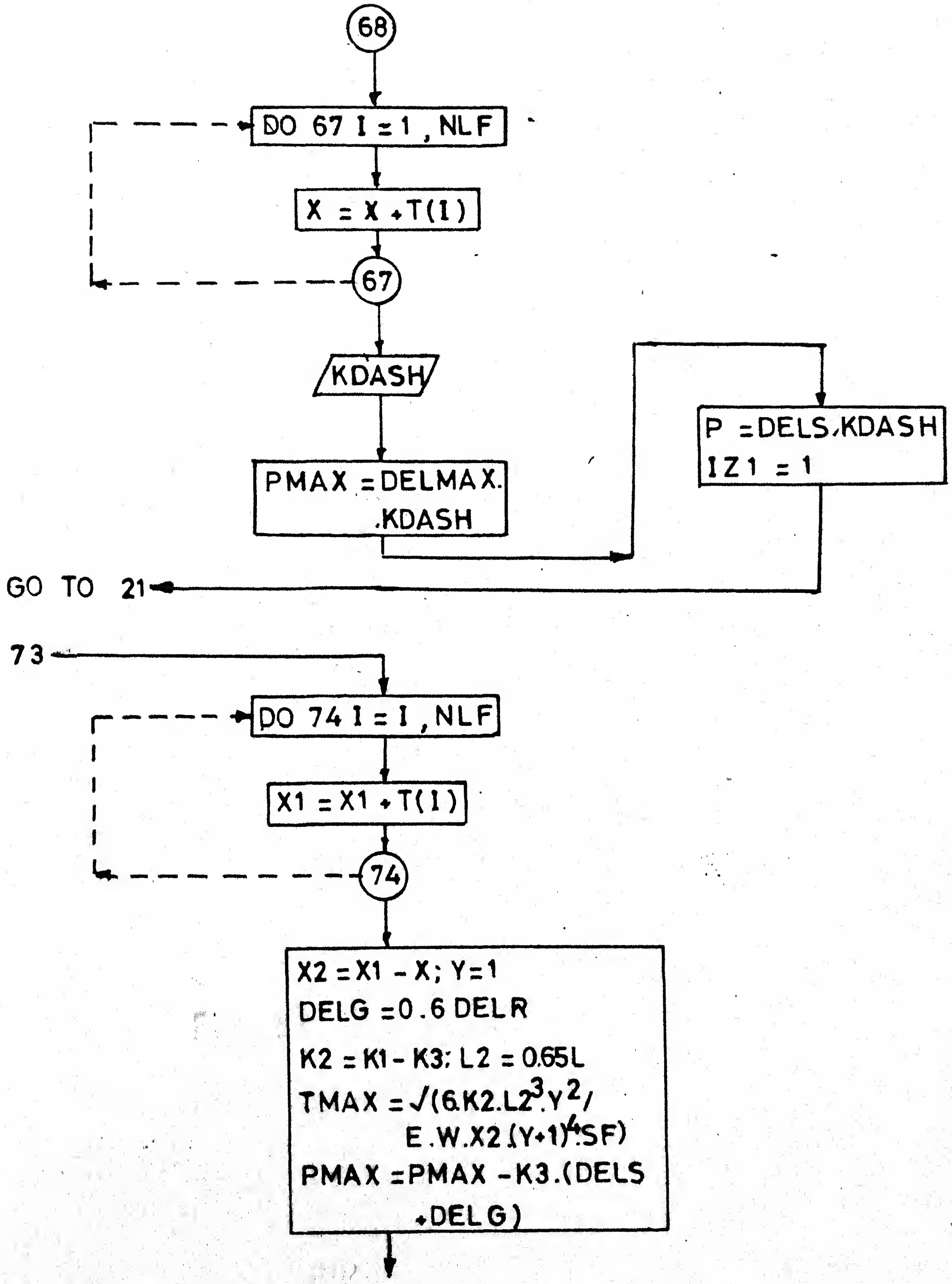



Fig. 3.2 Variable rate leaf spring design.

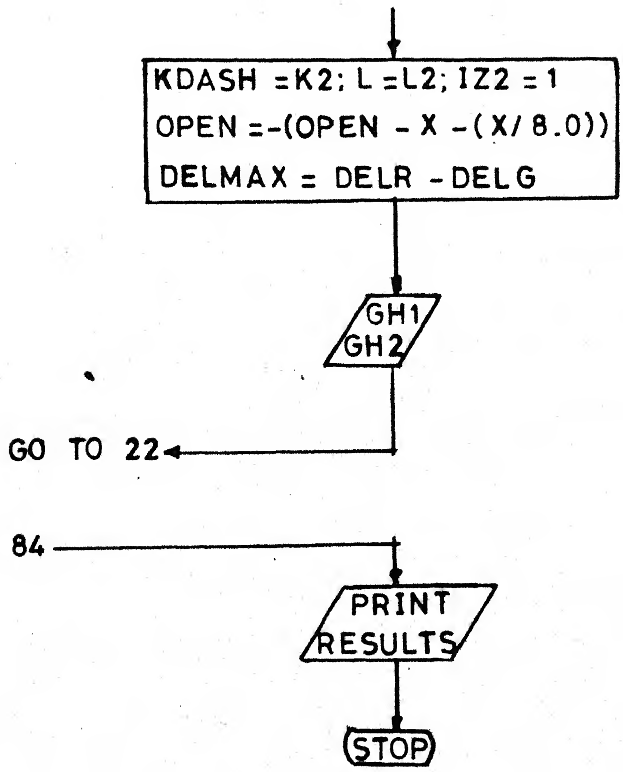



Fig 3.2 Continued

$$NSET\ (NS) = \frac{SIGI - ASUM}{IMOM(NS)}$$

where,

NSET (NS) - Number of leaves in $NS^{th}$ set.

SIGI - Total moment of inertia of the spring.

ASUM - Moment of inertia of the spring for (NS-1) sets.

IMOM(NS) - Moment of inertia of one leaf in the $NS^{th}$ set.

The flow chart is as shown in Fig. 3.3

### 3.4.2 Subroutine STRESS:

This calculates the stress in the leaf spring, under design or maximum load as the case may be

$$STS = \frac{(A + B)\ .\ TMAX}{2\ .\ TOTMI} \cdot \frac{DUM}{FAC}$$

where,

STS - Stress

A
B - Leaf overhangs

TMAX - Thickness of main leaf

DUM - Design or maximum load as the case may be

TOTMI - Total moment of inertia

FAC - $\dfrac{Y}{(Y+1)^2}$

Y - B/A

The flow chart is as shown in Fig. 3.4

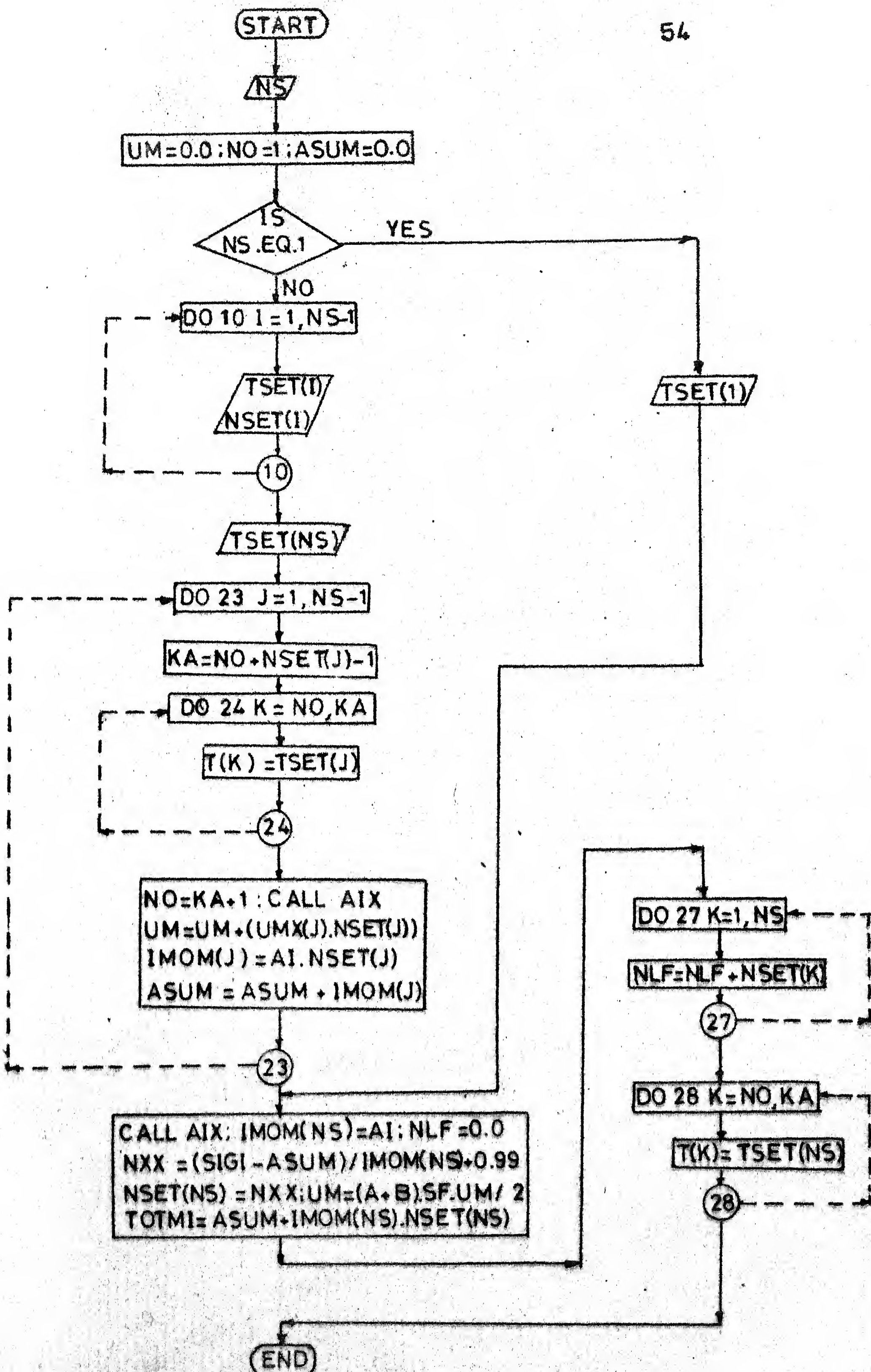



Fig. 3.3 Subroutine LFCOMB

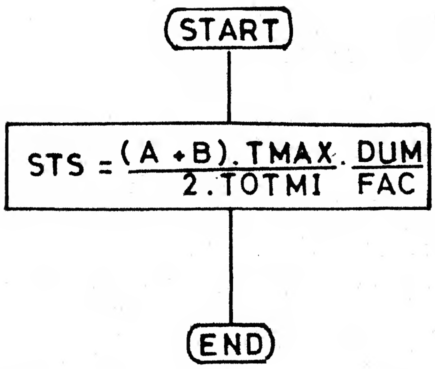



Fig. 3.4 Subroutine STRESS

### 3.4.3 Subroutine ASSTR:

This subroutine calculates the assembly stresses required in each leaf. This is a trial and error method as explained in Section 2.1.4.

### 3.4.4 Subroutine LEAFRA:

This subroutine calculates the unassembled or free radii of leaves.

$$\text{UNASRA} = \left( \frac{1}{\frac{1}{\text{ASRA}} - \frac{2.0 \,.\, \text{ASYST}}{\text{E.T}}} \right)$$

where,

| | | |
|---|---|---|
| ASYST | - | Assembly stress. |
| ASRA | - | Assembled radius of curvature. |
| UNASRA | - | Free radius of curvature |
| E | - | Young's modulus. |
| T | - | Thickness. |

The flow chart is as shown in Fig. 3.5.

### 3.4.5 Subroutine OVHANG:

This subroutine calculates the overhang or step of each leaf.

$$H\,(I) = \frac{L}{2} \cdot \frac{\text{ASYST}\,(I),\ (T(I))^2}{\sum\limits_{I=1}^{\text{NLF}} \text{ASYST}(I).\ (T(I))^2}$$

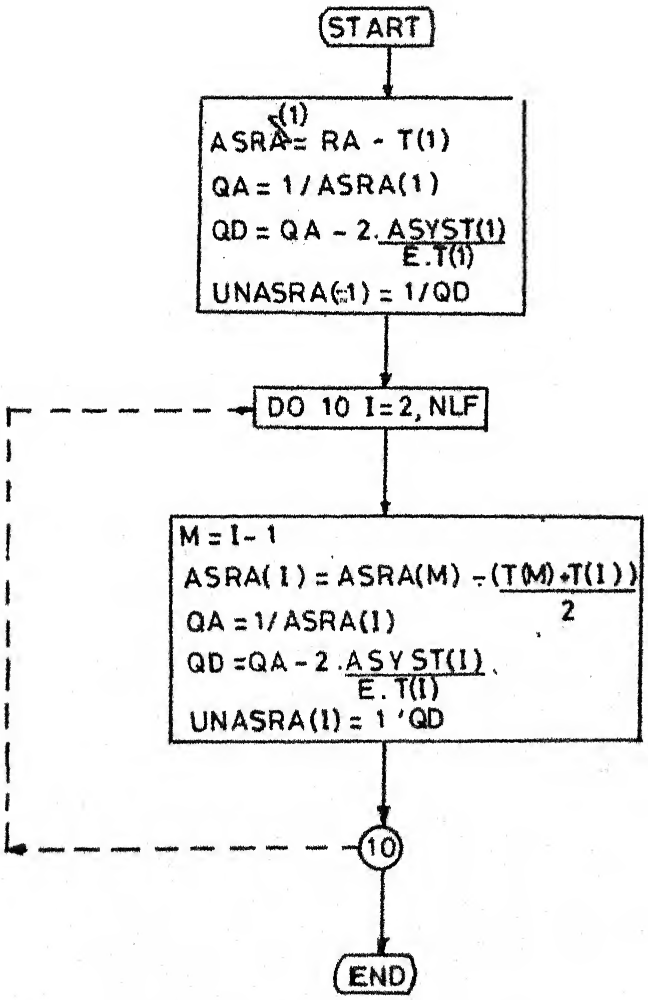



Fig.3.5 Subroutine LEAFRA

where,

H(I) - Overhang of $I^{th}$ leaf.

L - Leaf length

NLF - Number of leaves

The flow chart is as shown in Fig. 3.6.

3.4.6 <u>Subroutine LEAFLN</u>:

This subroutine calculates the length of each leaf. The leaf lengths are obtained by adding the successive overhangs. For the shortest leaf, the inactive portion, i.e. (ALNIN) lining must be added. It must also be added with 2(BED). The flow chart is shown in Fig. 3.7.

3.4.7 <u>Subroutine ENWT</u>:

This subroutine calculates the energy stored per unit weight and the total weight of the spring.

Strain energy per unit volume is,

$$= \frac{(STS)^2}{6\ E}$$

The flow chart is as shown in Fig. 3.8.

3.4.8 <u>Subroutine CAMBER</u>:

This subroutine calculates the camber of each leaf when unassembled or free.

$$CAM(I) = -\frac{(LF\ (I))^2}{80\ .\ UNASRA\ (I)}$$

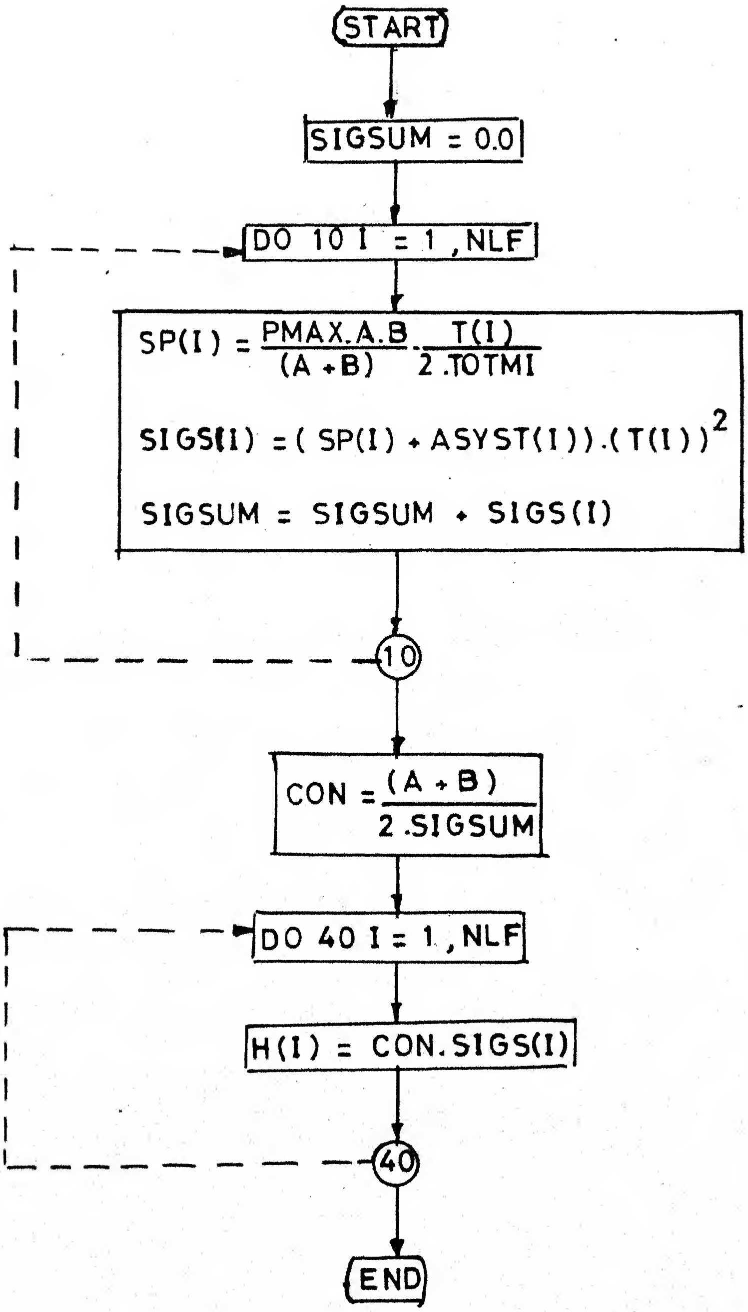



Fig.3.6 Subroutine OVHANG

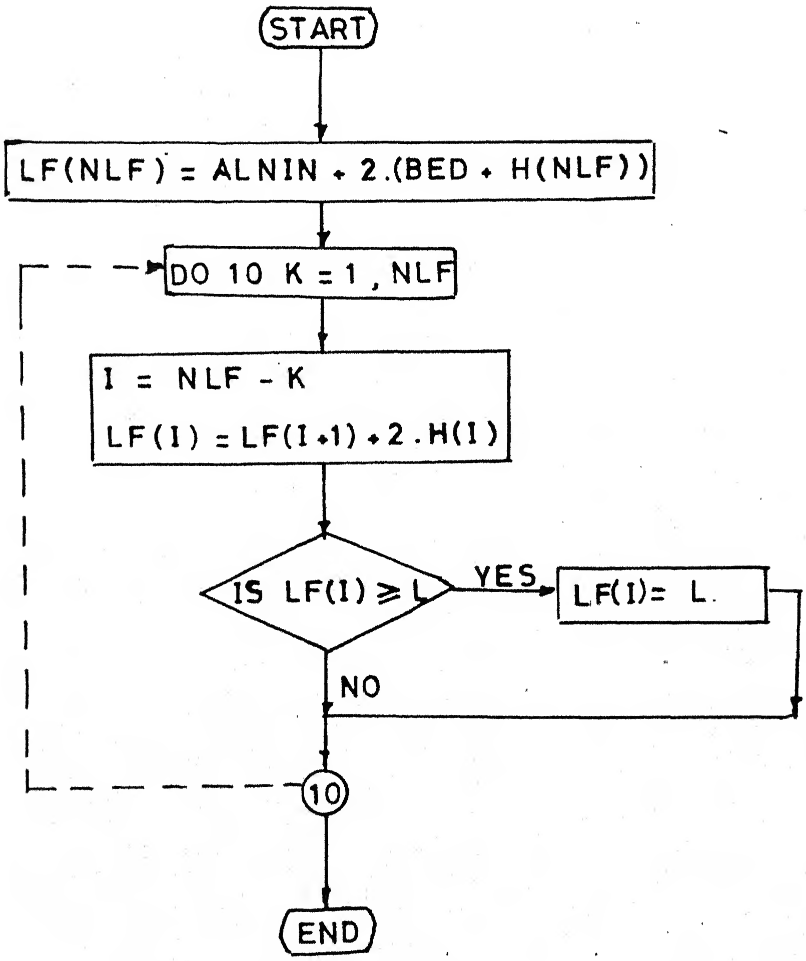



Fig 37 Subroutine LEAFLN

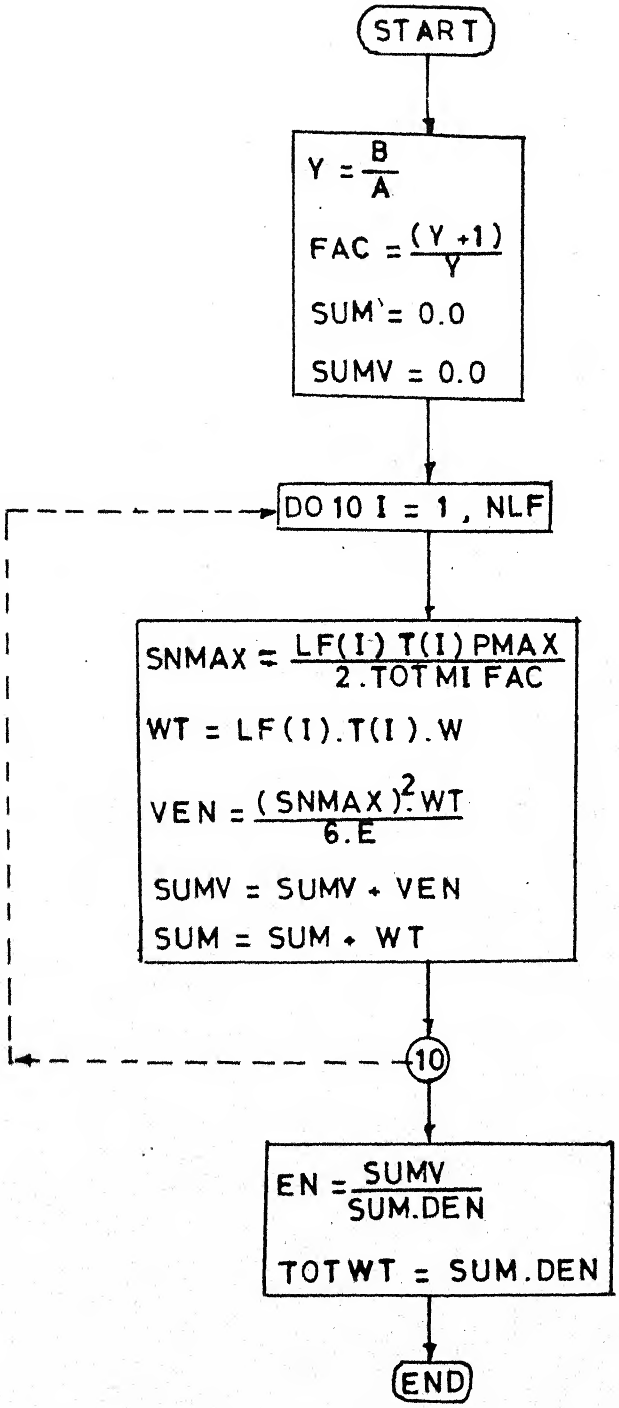



Fig. 3.8 Subroutine ENWT

where,

CAM (I) - Camber of $I^{th}$ leaf

LF (I) - Length of $I^{th}$ leaf.

The flow chart is shown in Fig. 3.9

### 3.4.9 Subroutine SPRWT:

This subroutine calculates the actual weight of spring after corrections have been made for spring length. The flow chart is shown in Fig. 3.10.

### 3.4.10 Subroutine STRLEF:

This subroutine calculates the stresses in each leaf for any given deflection. The flow chart is shown in Fig. 3.11.

Total stress = Load stress + Assembly stress.

### 3.4.11 Table Subroutines:

The subroutines for table of eye diameters, table for thicknesses of leaf, static deflection and ride clearances, stiffemning factors are also available.

## 3.5 Variables in the Program:

The variables in the program are listed below in alphabetical order.

A - Leaf overhang (front)

ALNIN - Lining or padding between leaves (= M+N).

ASCAM - Assembly camber.

ASRA - Assembly radius of curvature.

ASYST - Assembly stresses.

B - Leaf overhang (rear).

CAM - Camber of leaf.

DELMAX - Maximum deflection

DELR - Ride clearance

DELS - Static deflection

DEN - Density of Steel.

E - Young's modulus.

ED - Eye-pin diameter.

ENDCR - End cropping

EN - Energy per unit weight

ERR - Error in stress distribution.

H - Overhang or step.

K - Spring rate.

KDASH - Initial spring rate.

L - Length of leaf spring (= A+B).

LF - Leaf lengths - developed.

M - Lining in front cantilever.

N - Lining in rear cantilever.

NLF - Number of leaves.

NS - Number of sets of leaves.

OPEN - Opening.

P - Design load.

PMAX - Maximum load.

QA - Curvature of assembly.

RA - Assembly radius of curvature.

S - Allowable stress.

SF - Stiffenning factor

SIGI - Sum of moment of inertia

STRDES - Design stress

STRMAX - Maximum stress

STS - Stress.

T - Thickness.

TOLLEN - Tolerance in length

TOLR - Tolerance in radius

TOLW - Tolerance in width.

TOTMI - Total moment of inertia.

TOTWT - Total weight of spring.

TSET - Thickness of leaf for NSET (NS).

UM - Approximate weight.

UNASRA - Unassembled radius.

W - Width.

Y - Cantilever ratio.

## 3.6 Programming Considerations:

The present program has been written in Fortran-10, developed and tested on DEC-1090 system of IIT Kanpur. The

program is code and unit independent. The tables of various parameters, that are implemented are according to Indian Standard. The program is made into several subroutines as described earlier in Section 3.4 in such a way that each subroutine calculates a specific parameter and for the easy understanding of the reader.

The interactive features provide for the change of static deflection and ride clearance in the first stage. This affects the spring rate.

The next stages of interaction can change the thickness parameter only. The thickness of individual leaf changing, the assembly stresses change. The camber of leaves also change. All these affect the total weight of the spring, the stresses in each leaf and the energy stored by the spring.

The interaction and the output results are stored in an output file, which can be printed out on a hard copy and kept for reference.

To aid in the main program developed, a few other programs were also developed. A program to give the axle center position for a given deflection of the spring was developed. The theory behind this are given in Section 2.3, Section 2.4, Section 2.5, Appendix II and Appendix III. The program was developed for both

the methods viz. center link extension method and two point deflection method. A small graphics program was developed which gives the specification of the spring that was developed in the main program. Another program was developed which gives the rate and assembly stresses in a spring. The input for this program were the leaf lengths, leaf thicknesses and camber of each leaf. All these programs have not been presented in this work.

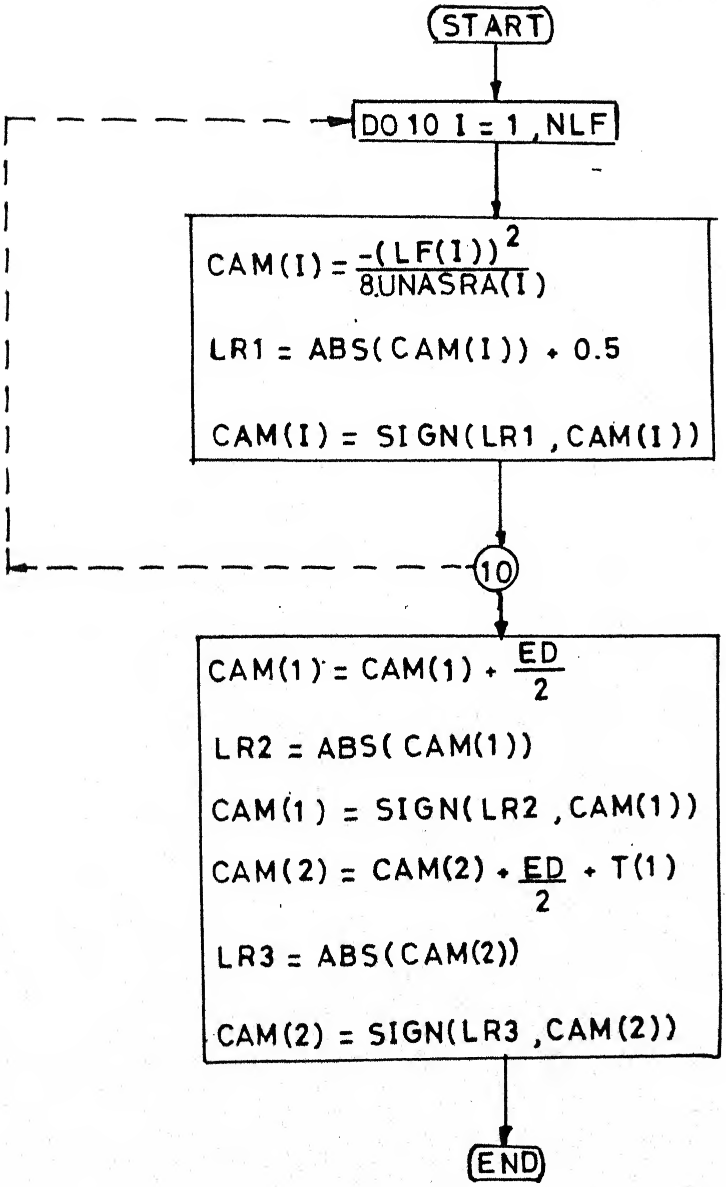



Fig. 3.9 Subroutine CAMBER

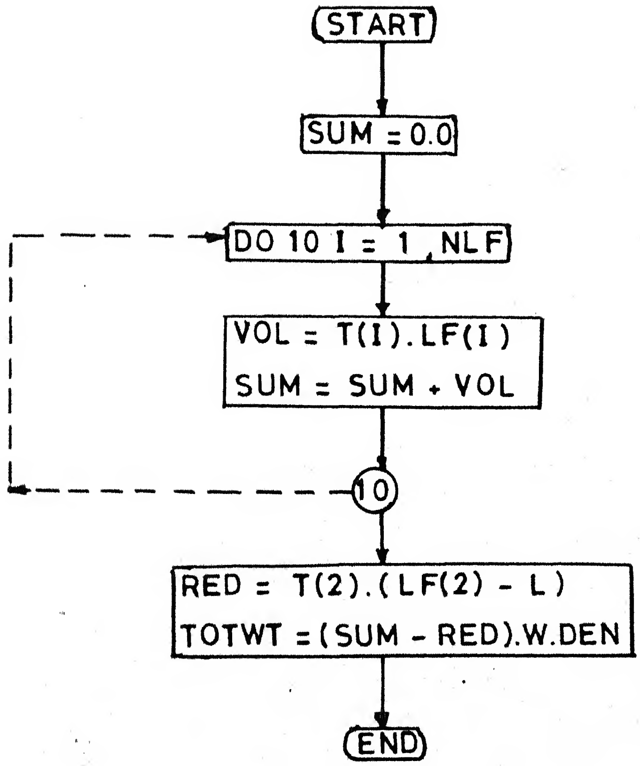



Fig. 3.10 Subroutine SPRWT

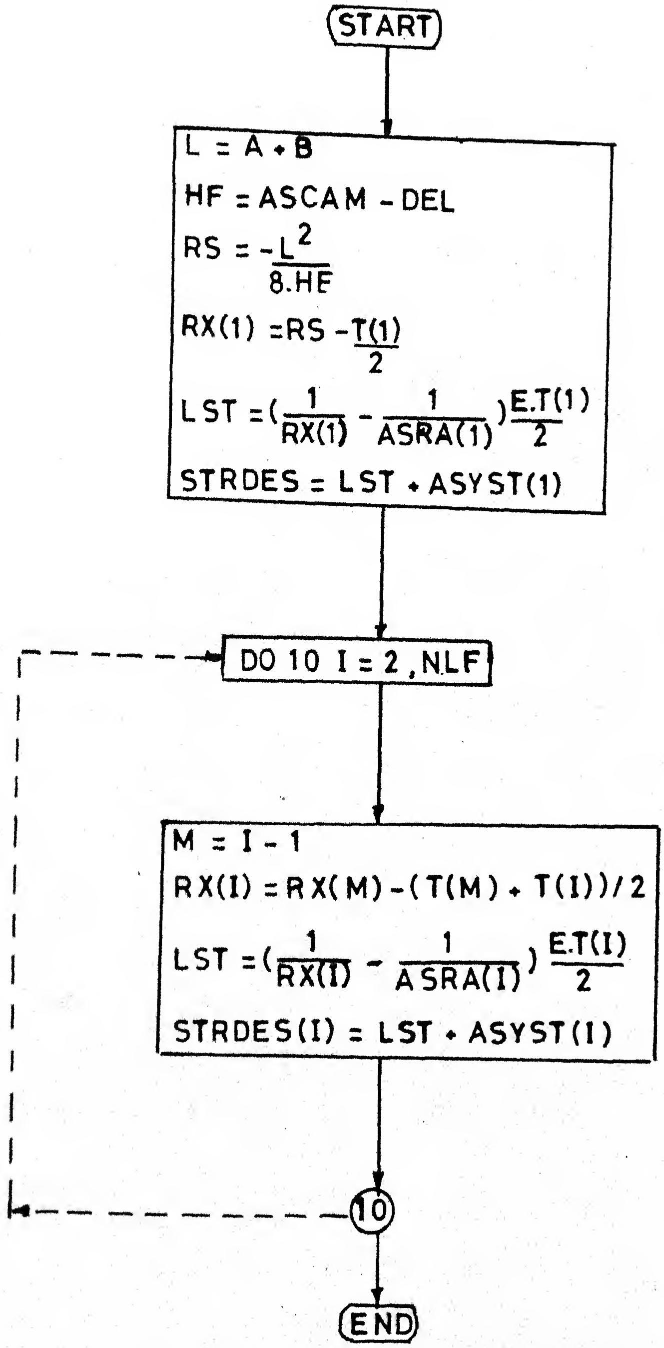



Fig. 3.11 Subroutine STRLEF

## CHAPTER - 4

## RESULTS AND DISCUSSIONS

### 4.1 Introduction and Data:

From the developed program a few examples have been solved and presented in this chapter. For the purpose of comparison, the same data is used for all these examples. Each example in this chapter shows a different type of design in the sense that in each case we have a different number of leaves, different thicknesses of leaves and corresponding analysis (analysis includes assembly stresses, design stresses and maximum stresses). The data for these problems are listed below.

| | |
|---|---|
| Load | 3470 kgf |
| Maximum allowable Stress | 86.15 $kgf/mm^2$ |
| Cantilever overhangs A | 800.0 mm |
| B | 800.0 mm |
| Width | 80.0 mm |
| Young's modulus of elasticity | 21000.0 $kgf/mm^2$ |
| Lining or seat length | 160.0 mm |
| Endcropping | 60.0 mm |
| Density | $0.785 \times 10^{-5}$ $kgf/mm^3$ |

| | |
|---|---|
| Opening or camber | 76.0 mm |
| Error in stress distribution | 2.0 kgf/mm$^2$ |
| Tolerances in radius | 20.0 mm |
| length | 0.8 mm |
| width | 0.2 mm |
| Static deflection | 142.0 mm |
| Ride clearance | 58.0 mm |
| Stiffenning factor | 1.1 |

## 4.2 Example 1:

In this example the thickness of all the leaves are kept constant. The number of leaves obtained is 16. The resulting assembly stresses, camber, leaf lengths and stress distribution are shown in Table 4.1. Fig. 4.1 shows stress distribution in leaves. The spring rate and weight of spring are obtained as 25.027 kgf/mm and 119.048 kgf respectively. The lowest stress is in the main leaf and its value is 33.610 kgf/mm$^2$. The highest stress is 58.777 kgf/mm$^2$ and is in the shortest leaf. The distribution of stresses in leaves is over 25.167 kgf/mm$^2$.

## 4.3 Example 2

In this example two sets of leaf thicknesses are used. The main leaf together with its adjacent leaf is made one gage thicker. The resulting assembly

stresses, camber, leaf lengths and stress distribution are shown in Table 4.2. The table also gives all the required parameters for design.

## 4.4 Example 3:

In this example a variable rate leaf spring is designed. The design procedure has been explained in Section 3.3. The values obtained for the first spring design is the same as that described in Section 4.2. The values obtained for the spring with higher rate are given in Table 4.3. The value obtained for the helper spring are given in Table 4.4. The variable rate leaf spring will be a combination of the first spring and the helper spring, which together give the same higher rate desired under heavy loads.

## 4.5 Examples 4, 5 and 6:

In these examples different thicknesses are used and the corresponding resultant stresses, lengths and figures are given.

## 4.6 Discussion:

In all the examples solved above the data used is the same. This gives the designer the option to choose whichever design that suits the job that is to be done. If the designer requires a spring of lesser weight or a spring that absorbs maximum total energy

or a spring that absorbs maximum energy per unit weight he can do so with the help of the program developed.

The weight of a spring reduces if there are less number of leaves. An increase in the thickness of the first two leaves to give sufficient strength to resist forces at the eye, will decrease the number of leaves and thus the weight of the spring. The strain energy stored in any leaf is given by,

$$\text{Strain energy} = \frac{\sigma^2}{6E} V$$

where,

$\sigma$ - stress in the leaf

$V$ - volume of the leaf

$E$ - Young's modulus of elasticity.

With the change of the number of leaves and the distribution of stresses the total strain energy stored changes. The desired stress distribution can be obtained with a combination of different leaf thicknesses. This fact can be seen from the following formula.

$$\sigma = \frac{Et}{2} \left( \frac{1}{R} - \frac{1}{R_o} \right)$$

where,

$t$ - thickness

$R$ - radius of spring when loaded (final)

$R_o$ - radius of spring when unloaded (initial)

The stress is proportional to thickness multiplied by change of curvature. For the same change of curvature the stress in any leaf is proportional to leaf thickness. The effect ofchange of leaf thickness will increase or decrease stress accordingly, as can be seen from Fig. 4.2

Table 4.1

Example 1.

| | |
|---|---|
| Spring rate required | 24.437 kgf/mm |
| Design load | 3470 kgf |
| Maximum load | 4887.3 kgf |
| Maximum permissible leaf thickness | 11.934 mm |
| Number of sets of leaves | 1 |
| Thickness of leaf | 11.0 mm |
| Number of leaves | 16 |
| Spring rate obtained | 25.027 kgf/mm |
| Weight of spring | 119.048 kgf |
| Energy per unit weight | 3.972 |

| Leaf No. | Thickness mm | Assembly stress $kgf/mm^2$ | Camber mm | Leaf lengths mm | Stresses at Design load $kgf/mm^2$ | Stresses at Maximum load $kgf/mm^2$ |
|---|---|---|---|---|---|---|
| 1 | 11.0 | -17.661 | 27.0 | 1872.0 | 33.610 | 54.624 |
| 2 | 11.0 | -12.118 | 58.0 | 1870.0 | 39.188 | 60.362 |
| 3 | 11.0 | - 8.347 | 38.0 | 1615.0 | 42.997 | 64.333 |
| 4 | 11.0 | - 5.421 | 42.0 | 1539.0 | 45.960 | 67.460 |
| 5 | 11.0 | - 3.035 | 42.0 | 1446.0 | 48.383 | 70.049 |
| 6 | 11.0 | - 1.044 | 41.0 | 1350.0 | 50.413 | 72.246 |
| 7 | 11.0 | 0.636 | 38.0 | 1251.0 | 52.132 | 74.136 |
| 8 | 11.0 | 2.059 | 34.0 | 1151.0 | 53.595 | 75.770 |
| 9 | 11.0 | 3.260 | 30.0 | 1048.0 | 54.836 | 77.186 |
| 10 | 11.0 | 4.264 | 25.0 | 944.0 | 55.881 | 78.407 |
| 11 | 11.0 | 5.089 | 20.0 | 838.0 | 56.749 | 79.453 |
| 12 | 11.0 | 5.750 | 16.0 | 732.0 | 57.451 | 80.336 |
| 13 | 11.0 | 6.254 | 12.0 | 624.0 | 57.999 | 81.666 |
| 14 | 11.0 | 6.610 | 8.0 | 516.0 | 58.399 | 81.651 |
| 15 | 11.0 | 6.822 | 5.0 | 408.0 | 58.655 | 82.094 |
| 16 | 11.0 | 6.892 | 3.0 | 299.0 | 58.771 | 82.399 |

Table 4.2

Example 2.

| | |
|---|---|
| Spring rate required | 24.437 kgf/mm |
| Design load | 3470 kgf |
| Maximum load | 4887.3 kgf |
| Maximum permissible leaf thickness | 11.934 mm |
| Number of sets of leaves | 2 |
| Thickness of leaf in I set | 13.0 mm |
| Number of leaves in I set | 2 |
| Thickness of leaf in II set | 11.0 mm |
| Number of leaves in II set | 13 |
| Spring rate obtained | 25.493 kgf/mm |
| Weight of spring | 110.824 kgf |
| Energy per unit weight | 4.181 |

| Leaf No. | Thickness | Assembly stress | Camber | Leaf lengths | Stresses at Design load | Stresses at Maximum load |
|---|---|---|---|---|---|---|
| | mm | $kgf/mm^2$ | mm | mm | $kgf/mm^2$ | $kgf/mm^2$ |
| 1 | 13.0 | -17.230 | 36.0 | 1891.0 | 43.367 | 68.218 |
| 2 | 13.0 | - 9.721 | 66.0 | 1888.0 | 50.926 | 76.002 |
| 3 | 11.0 | - 5.970 | 39.0 | 1525.0 | 45.386 | 66.782 |
| 4 | 11.0 | - 3.243 | 40.0 | 1418.0 | 48.152 | 69.712 |
| 5 | 11.0 | - 1.091 | 39.0 | 1324.0 | 50.341 | 72.067 |
| 6 | 11.0 | 0.663 | 37.0 | 1227.0 | 52.134 | 74.030 |
| 7 | 11.0 | 2.116 | 33.0 | 1128.0 | 53.626 | 75.692 |
| 8 | 11.0 | 3.323 | 29.0 | 1027.0 | 54.873 | 77.112 |
| 9 | 11.0 | 4.321 | 24.0 | 925.0 | 55.912 | 78.325 |
| 10 | 11.0 | 5.134 | 19.0 | 821.0 | 56.767 | 79.357 |
| 11 | 11.0 | 5.781 | 15.0 | 717.0 | 57.456 | 80.225 |
| 12 | 11.0 | 6.273 | 11.0 | 611.0 | 57.990 | 80.941 |
| 13 | 11.0 | 6.619 | 8.0 | 505.0 | 58.379 | 81.513 |
| 14 | 11.0 | 6.824 | 5.0 | 398.0 | 58.629 | 81.949 |
| 15 | 11.0 | 6.892 | 3.0 | 291.0 | 58.742 | 82.250 |

Table 4.3

Example 3.

| | |
|---|---|
| Spring rate required | 40.0 kgf/mm |
| Design load | 5680 kgf |
| Maximum load | 8000 kgf |
| Maximum permissible leaf thickness | 11.934 mm |
| Number of sets of leaves | 1 |
| Number of leaves | 26 |
| Spring rate obtained | 40.668 kgf/mm |
| Weight of spring | 194.284 kgf |
| Energy per unit weight | 4.074 |

| Leaf No. | Thickness mm | Assembly stress $kgf/mm^2$ | Camber mm | Leaf lengths mm | Stress at Design load $kgf/mm^2$ | Stress at Maximum load $kgf/mm^2$ |
|---|---|---|---|---|---|---|
| 1 | 11.0 | -19.384 | 22.0 | 1872.0 | 31.884 | 52.901 |
| 2 | 11.0 | -14.836 | 46.0 | 1870.0 | 36.471 | 57.645 |
| 3 | 11.0 | -11.771 | 28.0 | 1615.0 | 39.572 | 60.908 |
| 4 | 11.0 | - 9.354 | 35.0 | 1600.0 | 42.027 | 63.527 |
| 5 | 11.0 | - 7.333 | 40.0 | 1600.0 | 44.086 | 65.752 |
| 6 | 11.0 | - 5.590 | 43.0 | 1566.0 | 45.867 | 67.701 |
| 7 | 11.0 | - 4.060 | 44.0 | 1509.0 | 47.436 | 69.440 |
| 8 | 11.0 | - 2.700 | 43.0 | 1451.0 | 48.835 | 71.011 |
| 9 | 11.0 | - 1.484 | 42.0 | 1391.0 | 50.093 | 72.442 |
| 10 | 11.0 | - 0.389 | 41.0 | 1331.0 | 51.229 | 73.754 |
| 11 | 11.0 | 0.599 | 39.0 | 1270.0 | 52.258 | 74.962 |
| 12 | 11.0 | 1.490 | 36.0 | 1208.0 | 53.192 | 76.077 |
| 13 | 11.0 | 2.296 | 34.0 | 1145.0 | 54.041 | 77.108 |
| 14 | 11.0 | 3.021 | 31.0 | 1082.0 | 54.810 | 78.062 |
| 15 | 11.0 | 3.673 | 28.0 | 1018.0 | 55.507 | 78.946 |
| 16 | 11.0 | 4.256 | 25.0 | 953.0 | 56.135 | 79.764 |
| 17 | 11.0 | 4.775 | 22.0 | 889.0 | 56.699 | 80.520 |
| 18 | 11.0 | 5.231 | 19.0 | 823.0 | 57.202 | 81.217 |
| 19 | 11.0 | 5.628 | 17.0 | 758.0 | 57.647 | 81.858 |
| 20 | 11.0 | 5.968 | 14.0 | 692.0 | 58.035 | 82.446 |
| 21 | 11.0 | 6.253 | 12.0 | 625.0 | 58.369 | 82.981 |
| 22 | 11.0 | 6.485 | 9.0 | 559.0 | 58.650 | 83.466 |
| 23 | 11.0 | 6.663 | 7.0 | 492.0 | 58.879 | 83.902 |
| 24 | 11.0 | 6.792 | 5.0 | 425.0 | 59.057 | 84.290 |
| 25 | 11.0 | 6.867 | 4.0 | 359.0 | 59.184 | 84.630 |
| 26 | 11.0 | 6.892 | 3.0 | 292.0 | 59.262 | 84.923 |

Table 4.4

Example 3 (for helper spring)

| | |
|---|---|
| Spring rate required | 14.973 kgf/mm |
| Maximum permissible leaf thickness | 8.336 mm |
| Number of sets of leaves | 1 |
| Number of leaves | 16 |
| Spring rate obtained | 15.771 kgf/mm |
| Weight of spring | 85.194 kgf |
| Energy per unit weight | 5.171 |

| Leaf No. | Thickness mm | Camber mm | Leaf length mm | Stress at max. load $kgf/mm^2$ |
|---|---|---|---|---|
| 1 | 8.5 | 122 | 1440 | 14.404 |
| 2 | 8.5 | 130 | 1440 | 14.310 |
| 3 | 8.5 | 121 | 1440 | 14.216 |
| 4 | 8.5 | 120 | 1440 | 14.123 |
| 5 | 8.5 | 113 | 1400 | 14.031 |
| 6 | 8.5 | 97 | 1300 | 13.940 |
| 7 | 8.5 | 83 | 1200 | 13.850 |
| 8 | 8.5 | 69 | 1100 | 13.760 |
| 9 | 8.5 | 57 | 1000 | 13.672 |
| 10 | 8.5 | 46 | 900 | 13.584 |
| 11 | 8.5 | 36 | 800 | 13.498 |
| 12 | 8.5 | 28 | 700 | 13.412 |
| 13 | 8.5 | 20 | 600 | 13.327 |
| 14 | 8.5 | 14 | 500 | 13.242 |
| 15 | 8.5 | 9 | 400 | 13.159 |
| 16 | 8.5 | 5 | 300 | 13.076 |

Table 4.5

Example 4.

| | |
|---|---|
| Spring rate required | 40.0 kgf/mm |
| Design load | 5680 kgf |
| Maximum load | 8000 kgf |
| Maximum permissible leaf thickness | 11.934 mm |
| Number of sets of leaves | 2 |
| Thickness of leaf in I set | 13.0 mm |
| Number of leaves in I set | 2 |
| Thickness of leaf in II set | 12.0 mm |
| Number of leaves in II set | 18 |
| Spring rate obtained | 41.708 kgf/mm |
| Weight of spring | 162.34 kgf |
| Energy per unit weight | 4.698 |

| Leaf No. | Thick- ness | Assem- bly stress | Camber | Leaf lengths | Stresses at Design load | Stresses at Maximum load |
|---|---|---|---|---|---|---|
| | mm | $kgf/mm^2$ | mm | mm | $kgf/mm^2$ | $kgf/mm^2$ |
| 1 | 13.0 | -18.092 | 34 | 1891 | 42.505 | 67.357 |
| 2 | 13.0 | -12.720 | 59 | 1888 | 47.927 | 73.003 |
| 3 | 12.0 | - 9.313 | 37 | 1616 | 46.715 | 70.063 |
| 4 | 12.0 | - 6.693 | 42 | 1575 | 49.379 | 72.924 |
| 5 | 12.0 | - 4.545 | 43 | 1503 | 51.572 | 75.316 |
| 6 | 12.0 | - 2.729 | 43 | 1429 | 53.435 | 77.380 |
| 7 | 12.0 | - 1.165 | 41 | 1354 | 55.046 | 79.195 |
| 8 | 12.0 | 0.194 | 39 | 1277 | 56.453 | 80.809 |
| 9 | 12.0 | 1.381 | 36 | 1199 | 57.689 | 82.254 |
| 10 | 12.0 | 2.419 | 32 | 1120 | 58.777 | 83.554 |
| 11 | 12.0 | 3.325 | 29 | 1040 | 59.733 | 84.725 |
| 12 | 12.0 | 4.111 | 25 | 959 | 60.571 | 85.780 |
| 13 | 12.0 | 4.786 | 21 | 877 | 61.299 | 86.729 |
| 14 | 12.0 | 5.360 | 18 | 795 | 61.925 | 87.579 |
| 15 | 12.0 | 5.836 | 15 | 712 | 62.456 | 88.337 |
| 16 | 12.0 | 6.220 | 12 | 629 | 62.895 | 89.000 |
| 17 | 12.0 | 6.516 | 9 | 545 | 63.247 | 89.591 |
| 18 | 12.0 | 6.725 | 6 | 461 | 63.514 | 90.094 |
| 19 | 12.0 | 6.850 | 4 | 377 | 63.696 | 90.516 |
| 20 | 12.0 | 6.892 | 3 | 292 | 63.797 | 90.859 |

Table 4.6

Example 4 (for helper spring)

| | |
|---|---|
| Spring rate required | 14.973 kgf/mm |
| Maximum permissible leaf thickness | 7.196 mm |
| Number of sets of leaves | 1 |
| Number of leaves | 11 |
| Spring rate obtained | 15.940 kgf/mm |
| Weight of spring | 41.418 kgf |
| Energy per unit weight | 15.071 |

| Leaf No. | Thickness mm | Camber mm | Leaf length mm | Stress at max. load $kgf/mm^2$ |
|---|---|---|---|---|
| 1 | 7.0 | 122 | 1040 | 43.447 |
| 2 | 7.0 | 128 | 1040 | 43.082 |
| 3 | 7.0 | 120 | 1040 | 42.722 |
| 4 | 7.0 | 119 | 1040 | 42.367 |
| 5 | 7.0 | 119 | 1040 | 42.016 |
| 6 | 7.0 | 118 | 1040 | 41.671 |
| 7 | 7.0 | 93 | 927 | 41.330 |
| 8 | 7.0 | 66 | 782 | 40.993 |
| 9 | 7.0 | 43 | 636 | 40.661 |
| 10 | 7.0 | 26 | 491 | 40.333 |
| 11 | 7.0 | 13 | 345 | 40.009 |

## Table 4.7

Example 5 (for helper spring)

| | |
|---|---|
| Spring rate required | 14.973 kgf/mm |
| Maximum permissible leaf thickness | 7.196 mm |
| Number of sets of leaves | 1 |
| Number of leaves | 11 |
| Spring rate obtained | 15.34 kgf/mm |
| Weight of spring | 41.715 kgf |
| Energy per unit weight | 15.168 |

| Leaf No. | Thickness mm | Camber mm | Leaf length mm | Assembly stresses $kgf/mm^2$ | Stress at max. load $kgf/mm^2$ |
|---|---|---|---|---|---|
| 1 | 7.0 | 90 | 1040 | -17.230 | 26.217 |
| 2 | 7.0 | 110 | 1040 | -19.579 | 33.502 |
| 3 | 7.0 | 111 | 1040 | - 5.006 | 37.716 |
| 4 | 7.0 | 116 | 1040 | - 1.683 | 40.684 |
| 5 | 7.0 | 120 | 1040 | 0.863 | 42.880 |
| 6 | 7.0 | 123 | 1040 | 2.841 | 44.511 |
| 7 | 7.0 | 104 | 948 | 4.362 | 45.692 |
| 8 | 7.0 | 75 | 800 | 5.494 | 46.487 |
| 9 | 7.0 | 50 | 650 | 6.278 | 46.939 |
| 10 | 7.0 | 30 | 501 | 6.740 | 47.072 |
| 11 | 7.0 | 14 | 350 | 6.892 | 46.901 |

Table 4.8

Example 6 (for helper spring)

| | |
|---|---|
| Spring rate required | 14.973 kgf/mm |
| Maximum permissible leaf thickness | 7.196 mm |
| Number of sets of leaves | 2 |
| Thickness of leaf in I set | 7.5 mm |
| Number of leaves in I set | 2 |
| Thickness of leaf in II set | 7.0 mm |
| Number of leaves in II set | 8 |
| Spring rate obtained | 15.169 kgf/mm |
| Weight of spring | 38.113 kgf |
| Energy per unit weight | 17.345 |

| Leaf No. | Thick-ness mm | Camber mm | Leaf length mm | Assembly stresses $kgf/mm^2$ | Stress at max. load $kgf/mm^2$ |
|---|---|---|---|---|---|
| 1 | 7.5 | 92 | 1040 | -17.230 | 29.306 |
| 2 | 7.5 | 115 | 1040 | - 7.629 | 38.489 |
| 3 | 7.0 | 114 | 1040 | - 3.104 | 39.567 |
| 4 | 7.0 | 119 | 1040 | 0.028 | 42.344 |
| 5 | 7.0 | 123 | 1040 | 2.345 | 44.311 |
| 6 | 7.0 | 111 | 977 | 4.079 | 45.700 |
| 7 | 7.0 | 79 | 822 | 5.437 | 46.628 |
| 8 | 7.0 | 52 | 665 | 6.216 | 47.162 |
| 9 | 7.0 | 30 | 508 | 6.725 | 47.338 |
| 10 | 7.0 | 14 | 350 | 6.892 | 47.178 |

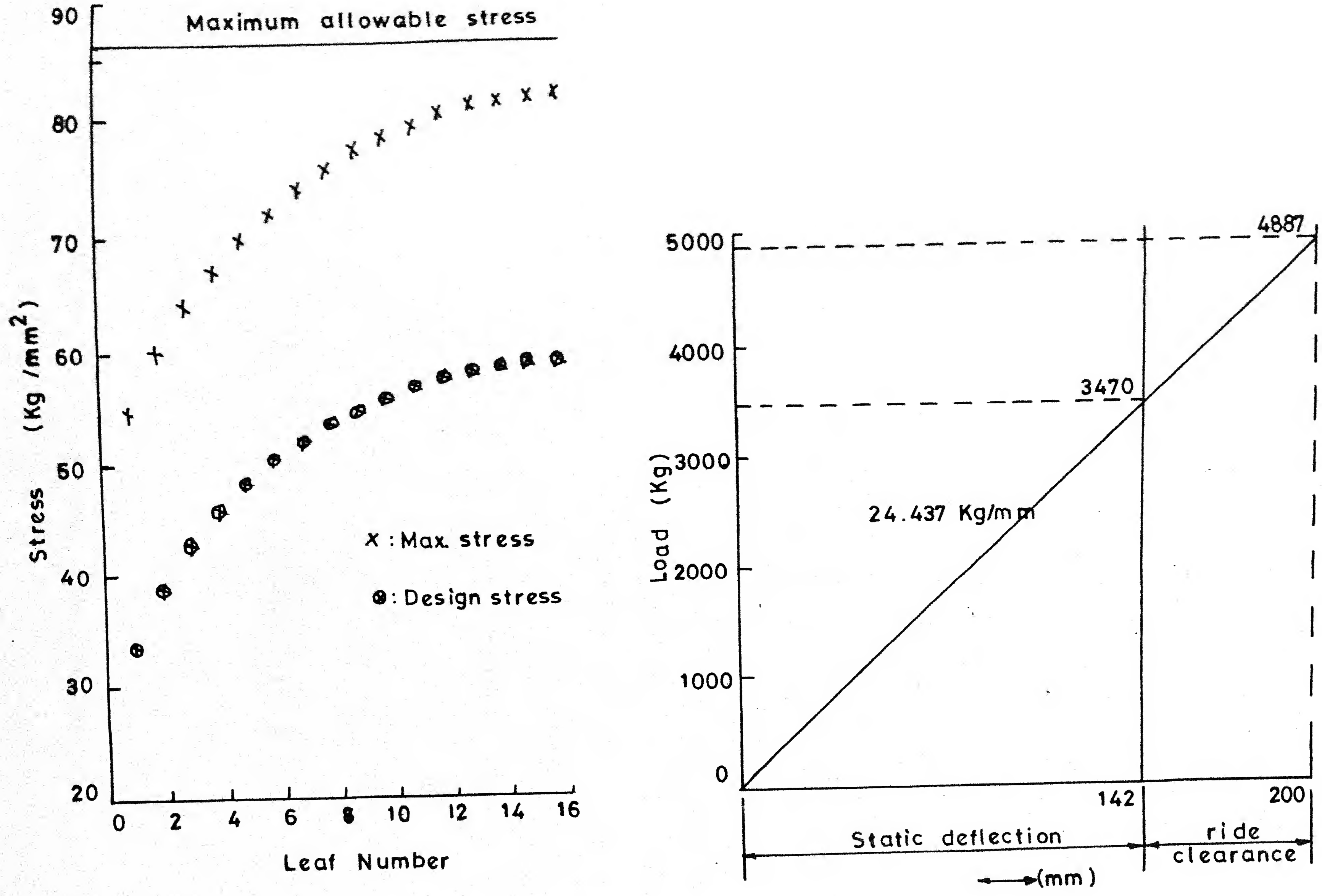



Fig. 4.1 Stress distribution and load rate diagram.

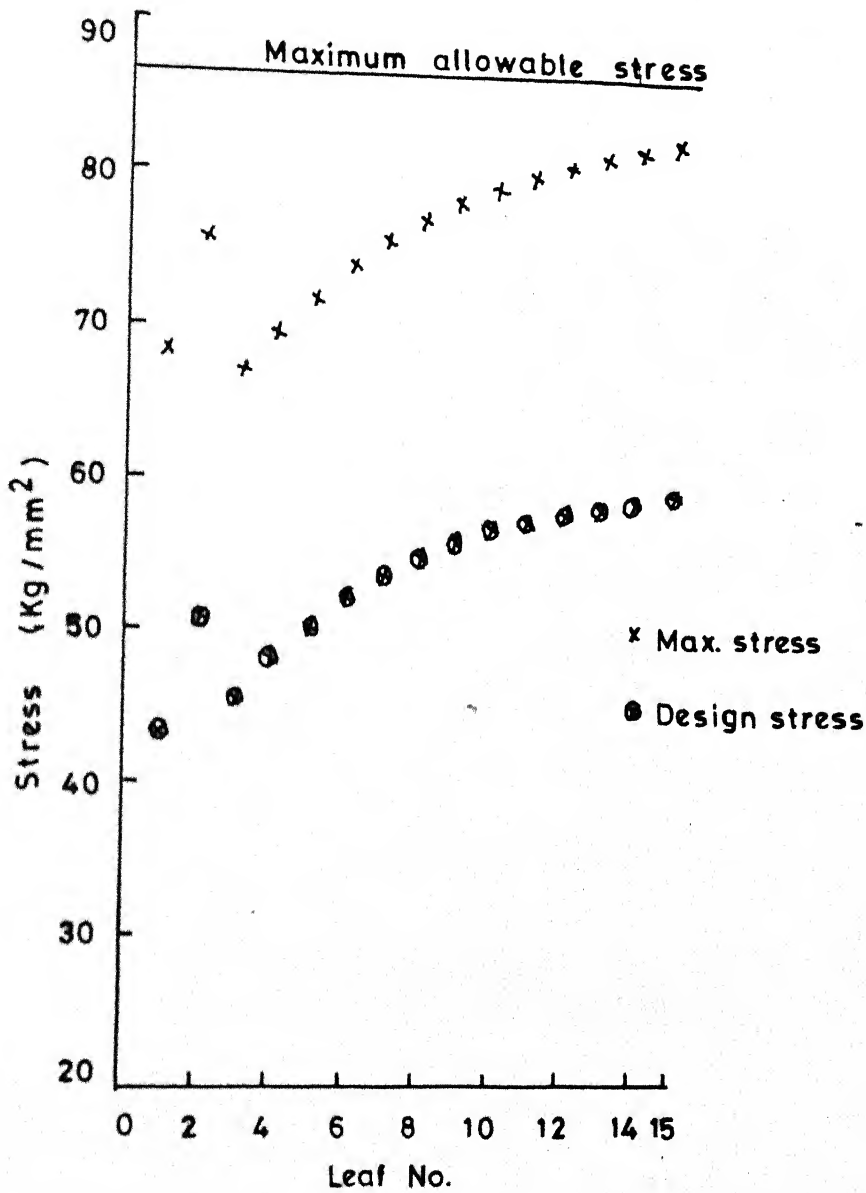
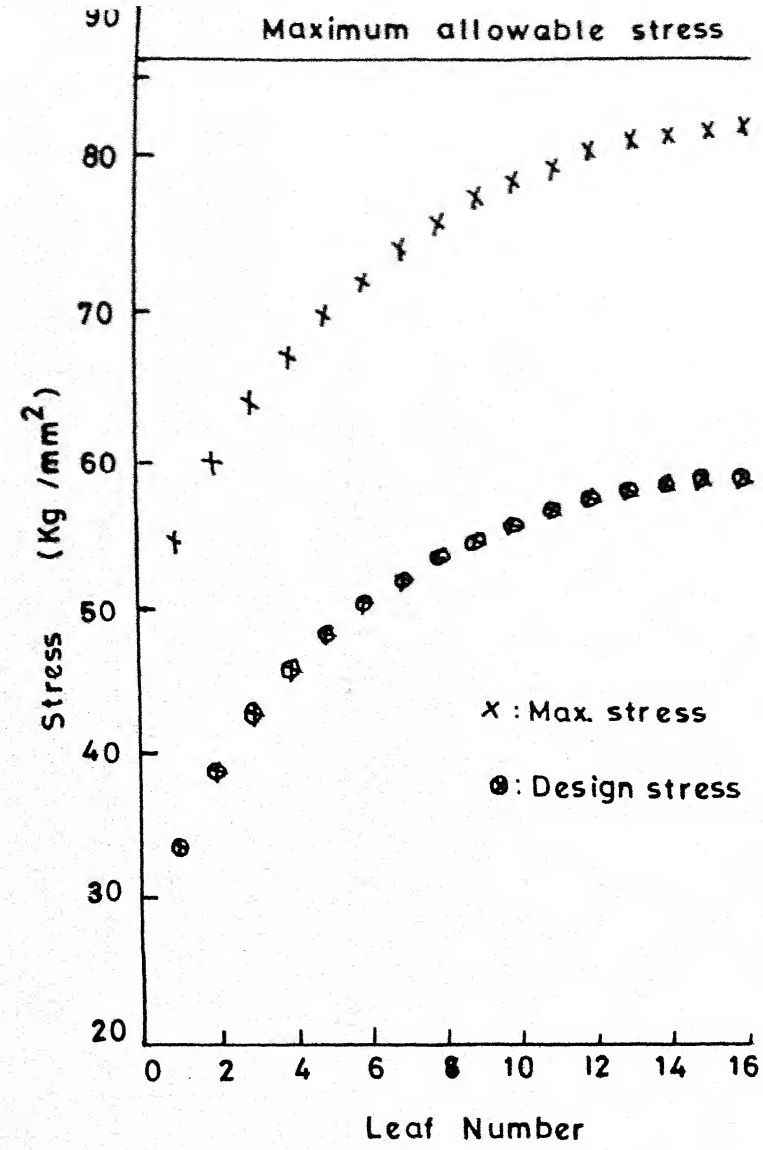



Fig. 4.2 Stress distribution -e.g. 2.



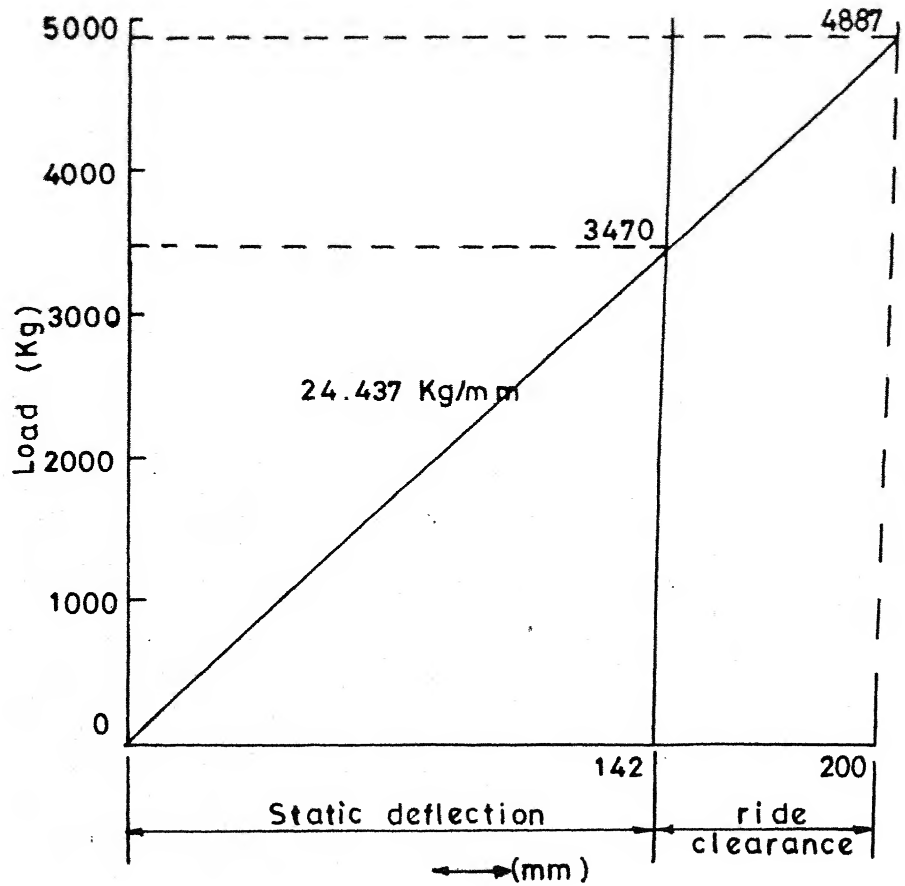




Fig. 4.3 Stress distribution and load rate diagram.

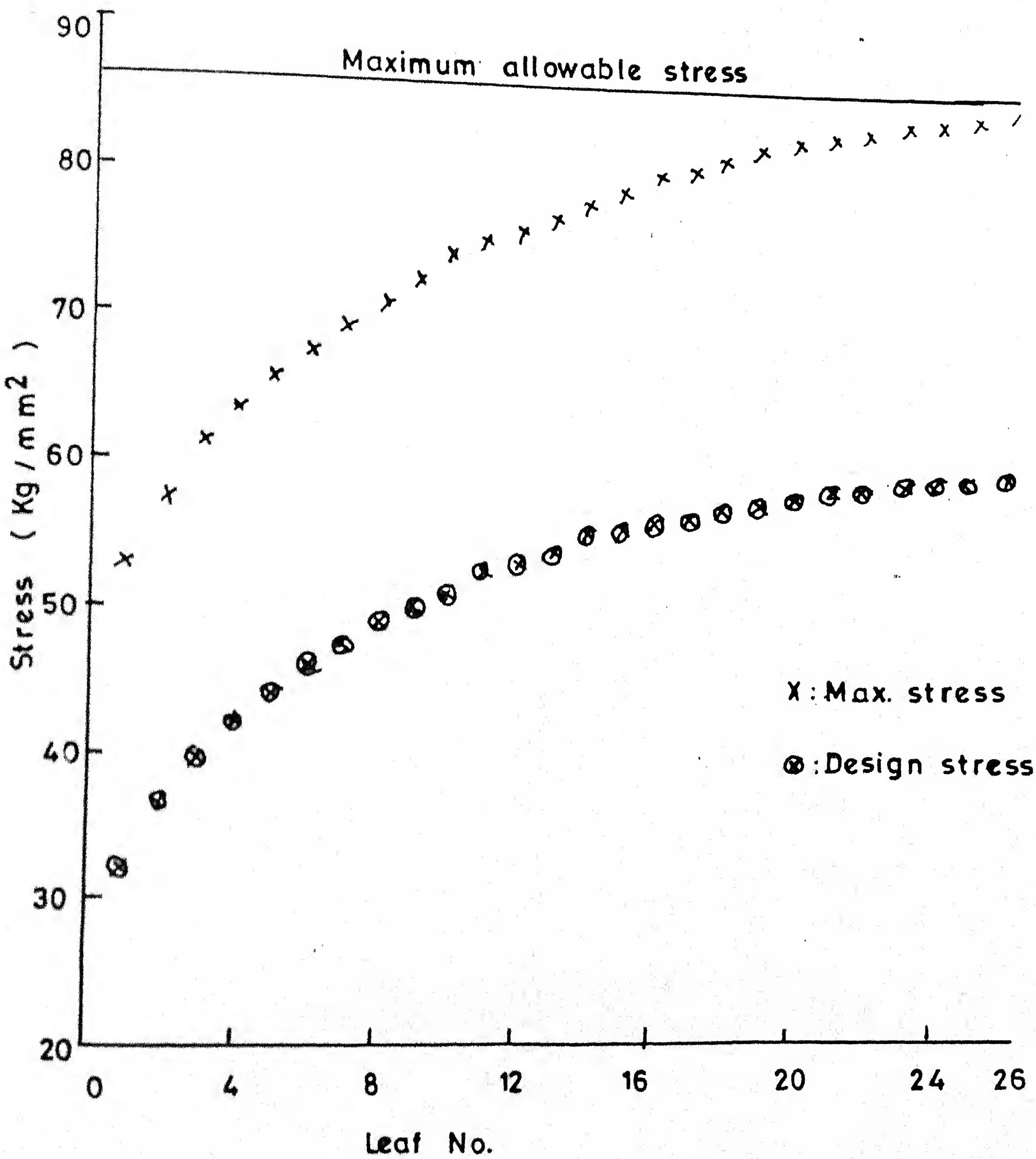
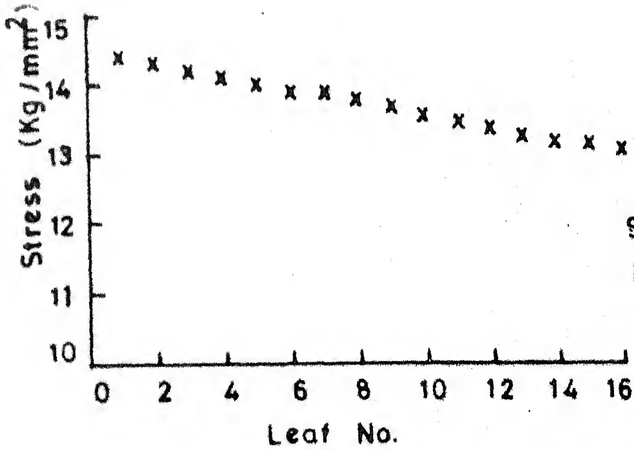
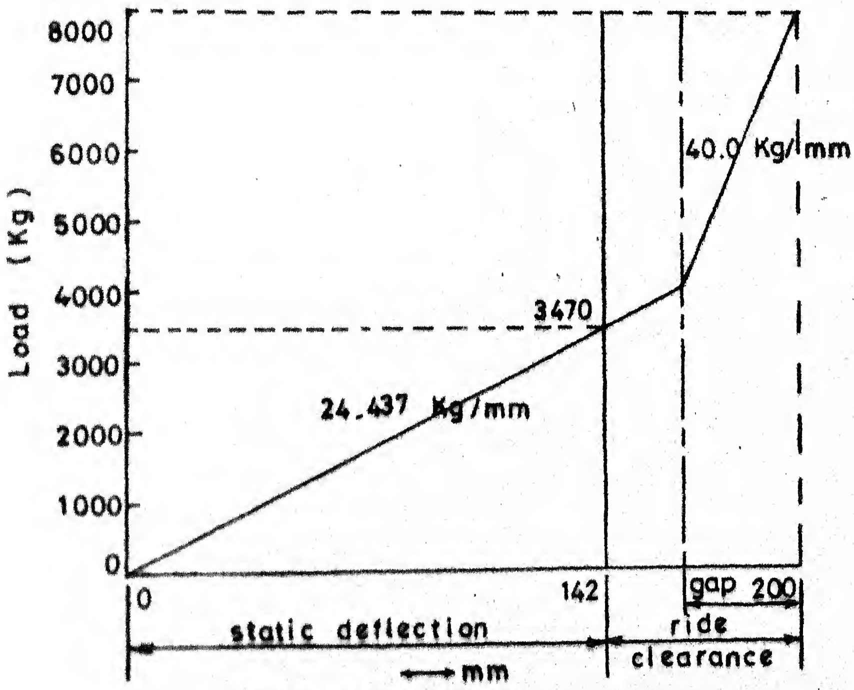



Fig. 4.4 Stress distribution e.g.3

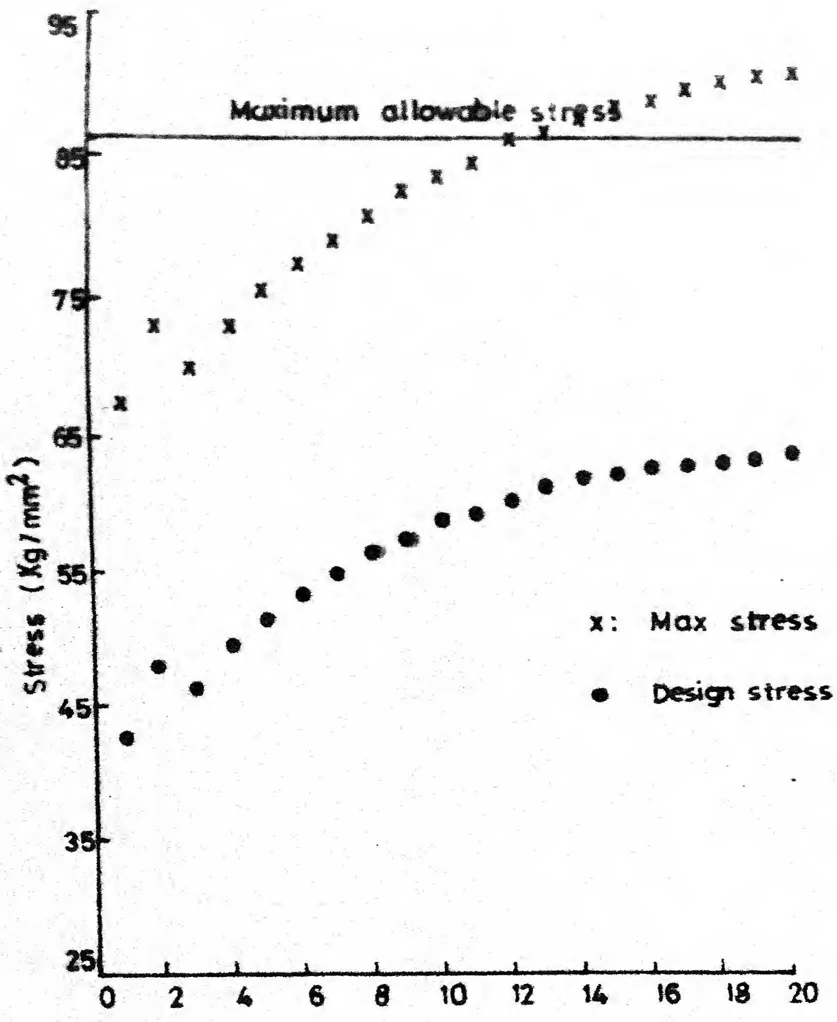



Stress distribution (for helper spring )

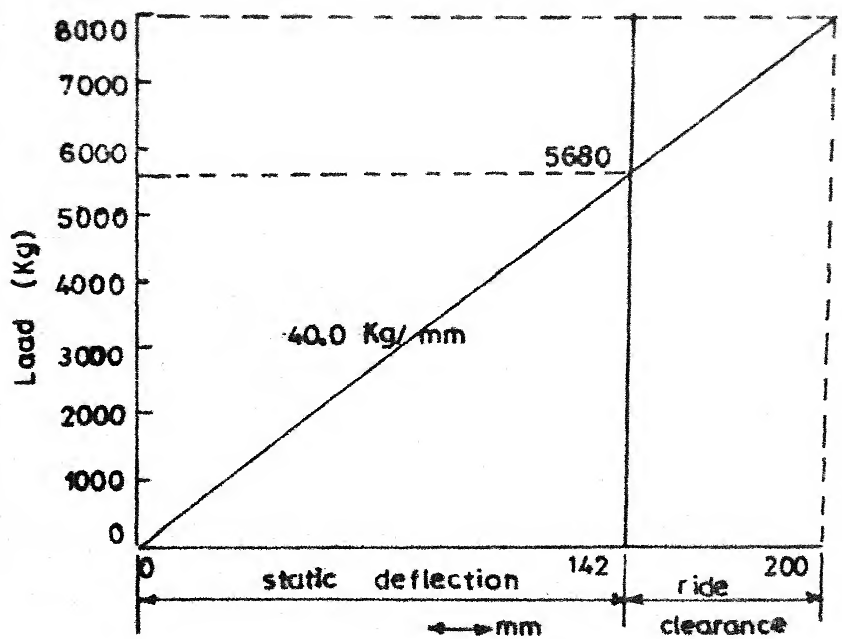



Load rate diagram

Fig.4.5 Example 3



Stress distribution



Load rate daigram

Fig. 4.6 Example 4

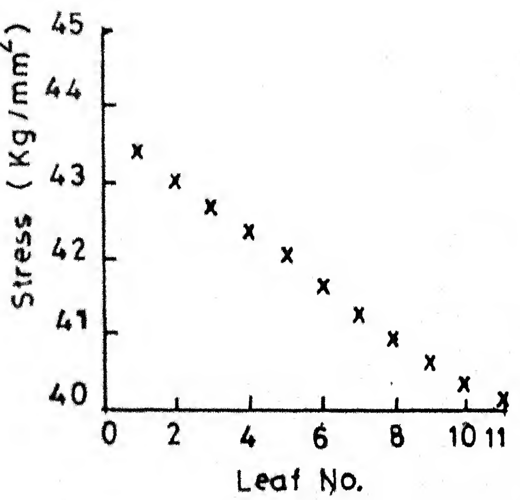
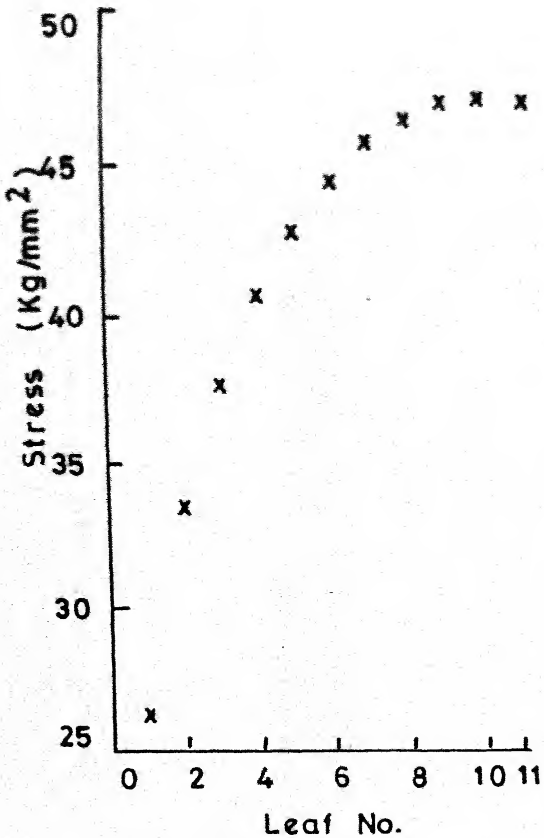
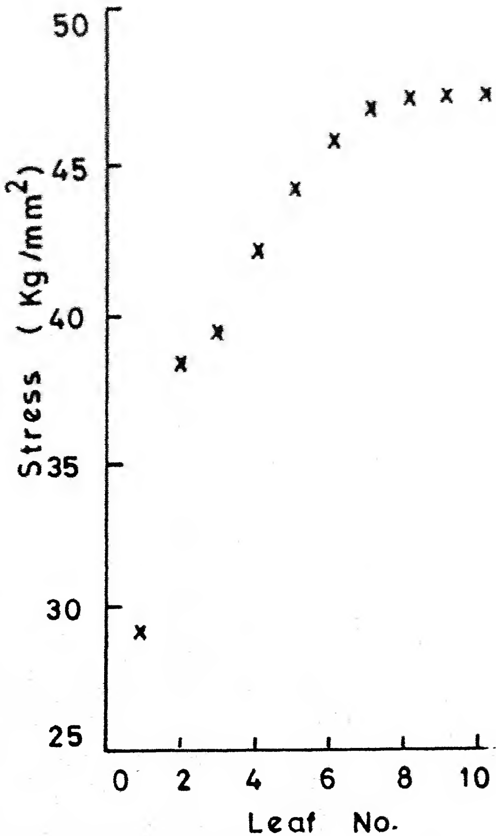



Fig. 4.7 Stress distribution (for helper spring) -e.g. 4.




Stress distribution (for helper spring)

Fig.4.8 Example 5

Fig.4.9 Example 6

# CHAPTER - 5

## CONCLUSIONS

### 5.1 Technical Summary:

The program for the computer aided design of leaf springs is developed and tested on DEC 1090 system of I.I.T. Kanpur. The program developed is unit and design-code independent. The program is interactive and user friendly. Anyone with a basic knowledge about leaf springs can use the program effectively.

The program provides for the selecting of a leaf combination to give the desired rate and weight of the spring. The user can interact with the program while in execution mode at this stage and change the leaf thicknesses and the number of sets of leaf thicknesses to obtain the desired rate and weight of the spring.

An important feature of this program is its capability to design a variable rate leaf spring with the help of a helper spring. The higher rate required under heavy load can be interactively changed to obtain the specified rate. As mentioned earlier in this

section, the program is unit independent. But in the program presented the Indian Standards are implemented for few parameters and the program reads load in kg. and deflection in mm. It can easily be changed over to other types by ignoring the units and feeding in the values of corresponding units.

## 5.2 Recommendations for Further Work:

A complete spring design for any application requires the design of bolts,clamps, alignment clips, eyes, spacers (in the case of variable rate leaf spring-helper type) along with the present design, which includes, leaf lengths, leaf cambers, leaf radii and leaf thicknesses. The present work comprises a portion of the complete design. The remaining portion also needs to be implemented in the program.

The present work can design a variable rate leaf spring of the helper type only. Other types of variable rate leaf springs such as multistage leaf spring (Fig. 2.5) or the use of curved bearing pads in achieving variable rate (Fig. 2.6) have to be incorporated into the program for a more generalised package.

A complete package on leaf springs will require development of a graphics program also. This graphics program should be such that it can draw the position of

the spring for any given deflection along with the load rate diagram and the stress distribution among leaves.

The present work has been verified and checked by manual efforts only. This has to be suplemented by sophisticated experimental setups. The measurement of stress and strain can be made by the effective use of micro-processors or/and strain gages. With these results to back the program, the present work could well become very powerful.

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# APPENDIX-I

## SPECIFICATIONS FOR A LEAF SPRING:

Most of the dimensions defined here refer to a datum line. It is the line along the lengthwise direction of the spring which passes through the centers of the eyes on springs having eyes. On other springs, it passes through the points where the load is applied near the ends of the spring.

Loaded Length: Distance between centers of the spring eyes when the spring is deflected to the specified load position. On spring without eyes, it is the distance between the lines along which load is applied near the ends, measured under the specified loading conditions. (Fig. A-2).

Loaded Fixed End Length: Distance from the center of the fixed eye to the projection on the datum line of the point where the center line of the center bolt inter - sects the spring surface in contact with the spring seat under specified loading conditions. (Fig. A-5).

Straight Length: Distance between spring eye centers when the main leaf is flat.

Finished Width: Width to which spring ends are finished to bring about flat bearing surfaces on the edges. (Distance A in Fig. A-3).

Leaf Numbers: Leaves are designated by numbers, starting with the main leaf which is No. 1. The adjoining leaf is No. 2, and so on. If auxiliary or refound leaves are used, the auxiliary leaf adjoining the main leaf is auxiliary leaf No. 1, the next one auxiliary leaf No. 2, and so on. (Auxiliary leaves are on that side of the main leaf on which load is applied to spring ends, away from the side from which load is applied to spring center). Helper springs are considered as separate units. (Fig.A-1)

Opening and Overall Height: Distance from the datum line to the point where the center bolt or cup center intersects the surface of the spring that is in contact with the spring seat.

If the surface in contact with the seat is on the main leaf or an auxiliary leaf (which is characteristic of 'under-slung' springs), this distance is called opening. (Fig. A-1)

If the surface in contact with the seat is on the shortest leaf (which is characteristic of overslung' springs), this distance is called overall height. (Fig. A-1).

Opening and overall height may be positive or negative.

Clearance: The difference in opening or overall height between the specified load position and the metal to metal contact position disregarding the rubber buffers is called clearance.

Seat Angle: It is the angle between the tangent to the center of the spring seat and line drawn through the terminal point of the active spring length at each eye taken along the tension surface of the main leaf (Fig. A-1). When both ends of the spring have eyes of identical configuration and diameter (or have plain ends without eyes), the seat angle is the angle between the tangent at the center of the spring seat and the datum line.

Seat Length: Length of the spring that is in physical engagement with the spring seat when installed on a vehicle at design height. It is always greater than the inactive or clamp length. (Fig. A-3, Fig. A-4, Fig. A-5).

Clamp Length: Length of spring rendered inactive by the clamp located on the side opposite the spring seat. It is always less than the length in physical engagement with the clamp. (Fig. A-3).

Camber: The distance from the datum line to the point where the center bolt or the cup center intersects the surface of the mainleaf. This may be either positive or negative. (Fig. A-1, Fig. A-5)

Curvature: Curvature (1/R) is the reciprocal of the radius (R). The curvature of a flat leaf is zero. Curvature is considered positive in the direction in which it increases with added load. Positive curvature corresponds to negative camber.

Load: Load is the force exerted by the spring at any specified camber or overall height or opening.

Load Rate: It is half the difference between the average of compression and release loads measured 25 mm above and 25 mm below the specified position and is expressed as kg/25 mm. Alternatively deflection rate can also be specified in terms of mm/100 kg.

Requirement Drawings: The specifications and requirement drawings for various types of leaf spring applications are shown in Fig. A-2, Fig. A-3, Fig. A-4, Fig. A-5 and Fig. 2.5. In all these cases in addition to what is specified in the figure the following must be specified:

a) Material
b) Hardness,
c) Spring shown under what load?
d) Rate of spring.

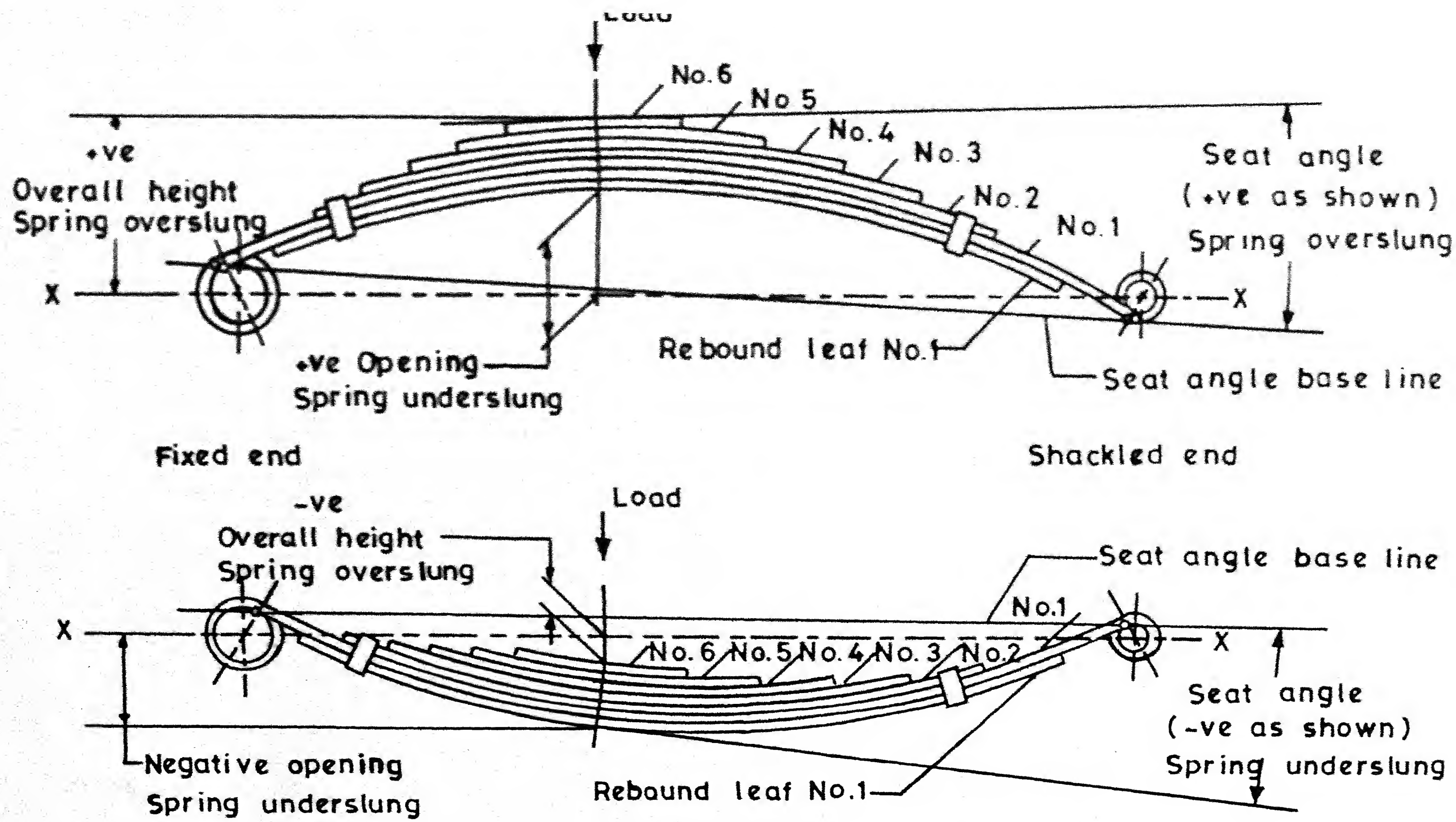



Fig.I.1 Measurement of opening, overall height and seat angle.

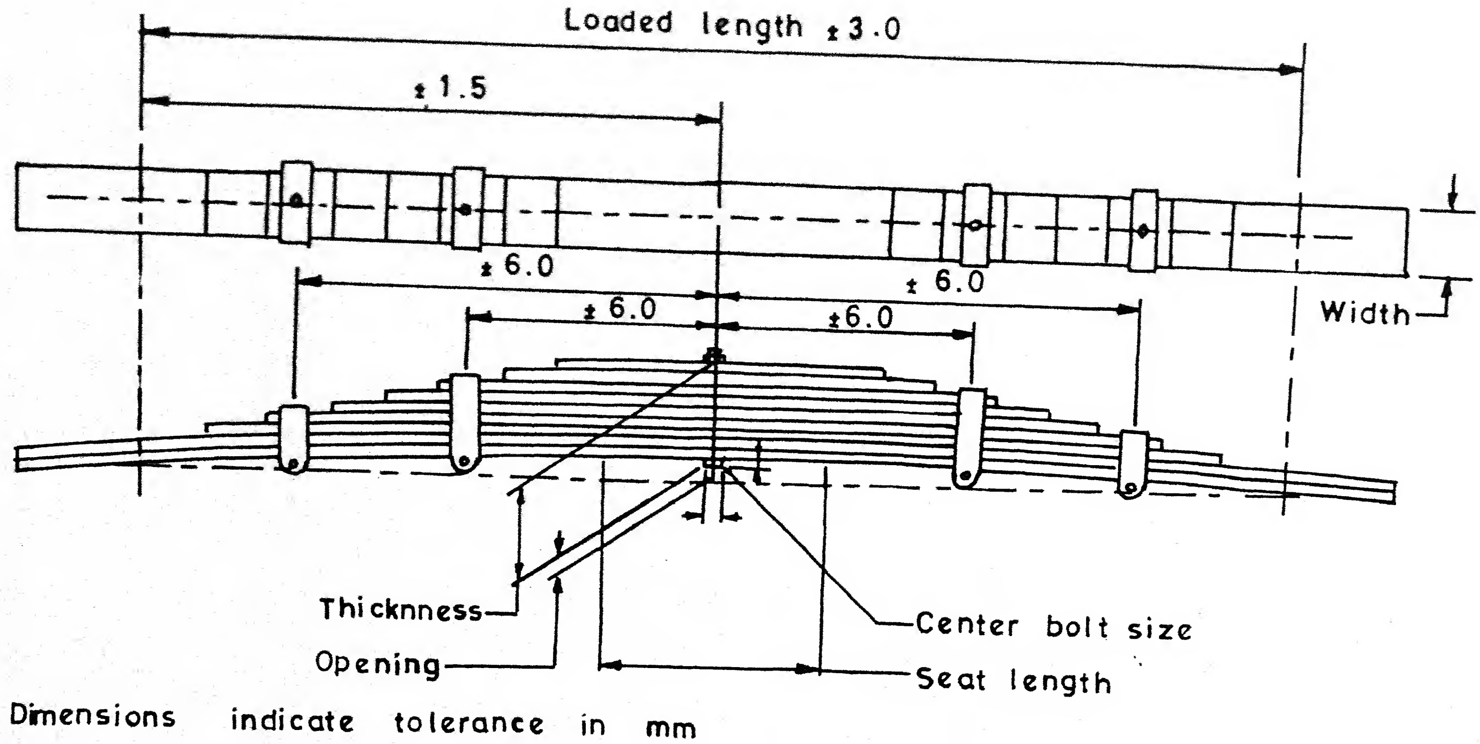
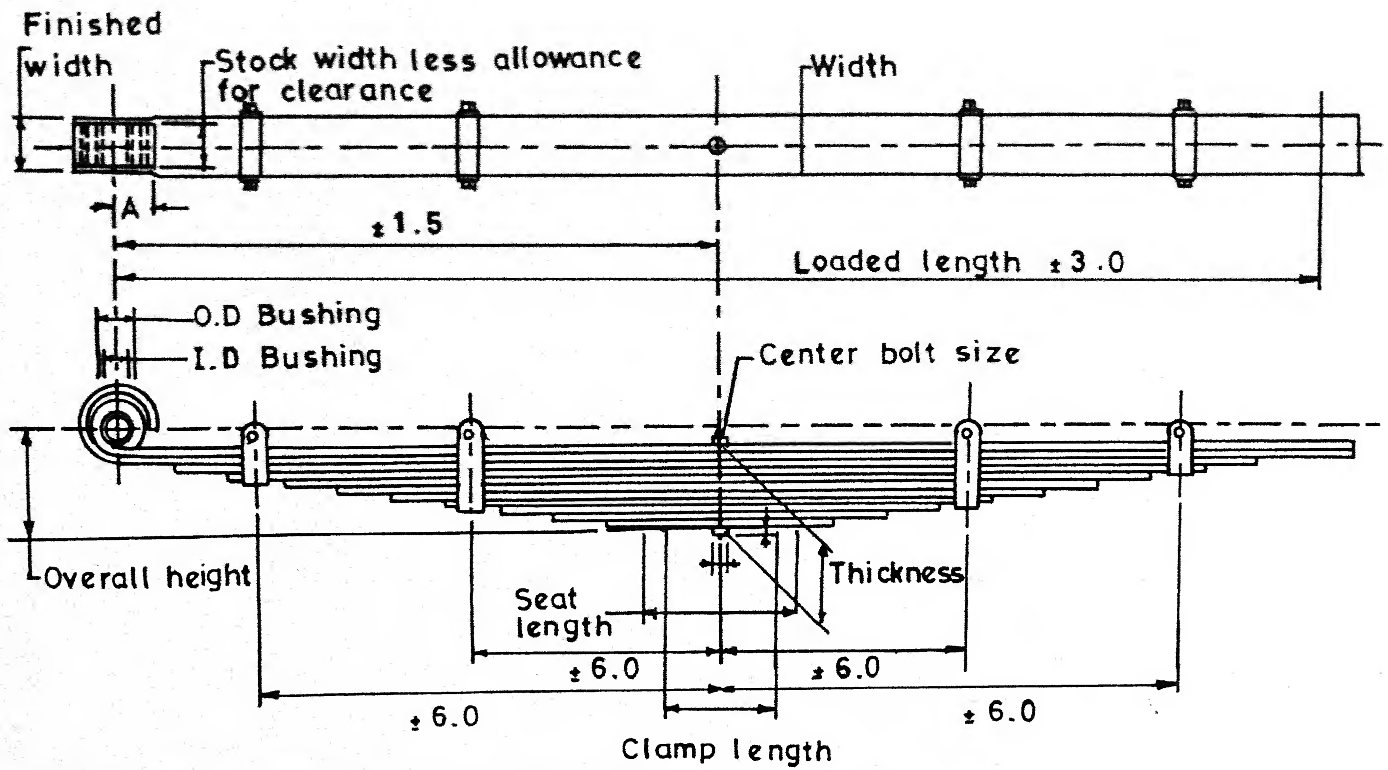
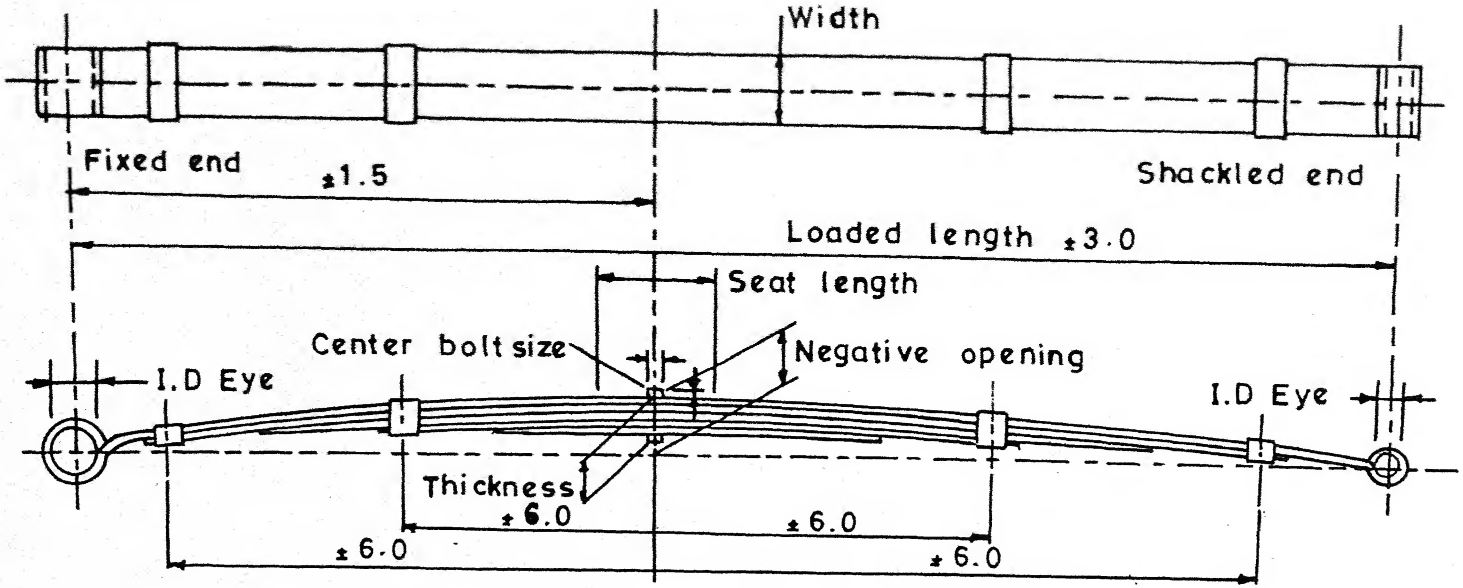



Fig. I.2 Spring with plain ends.



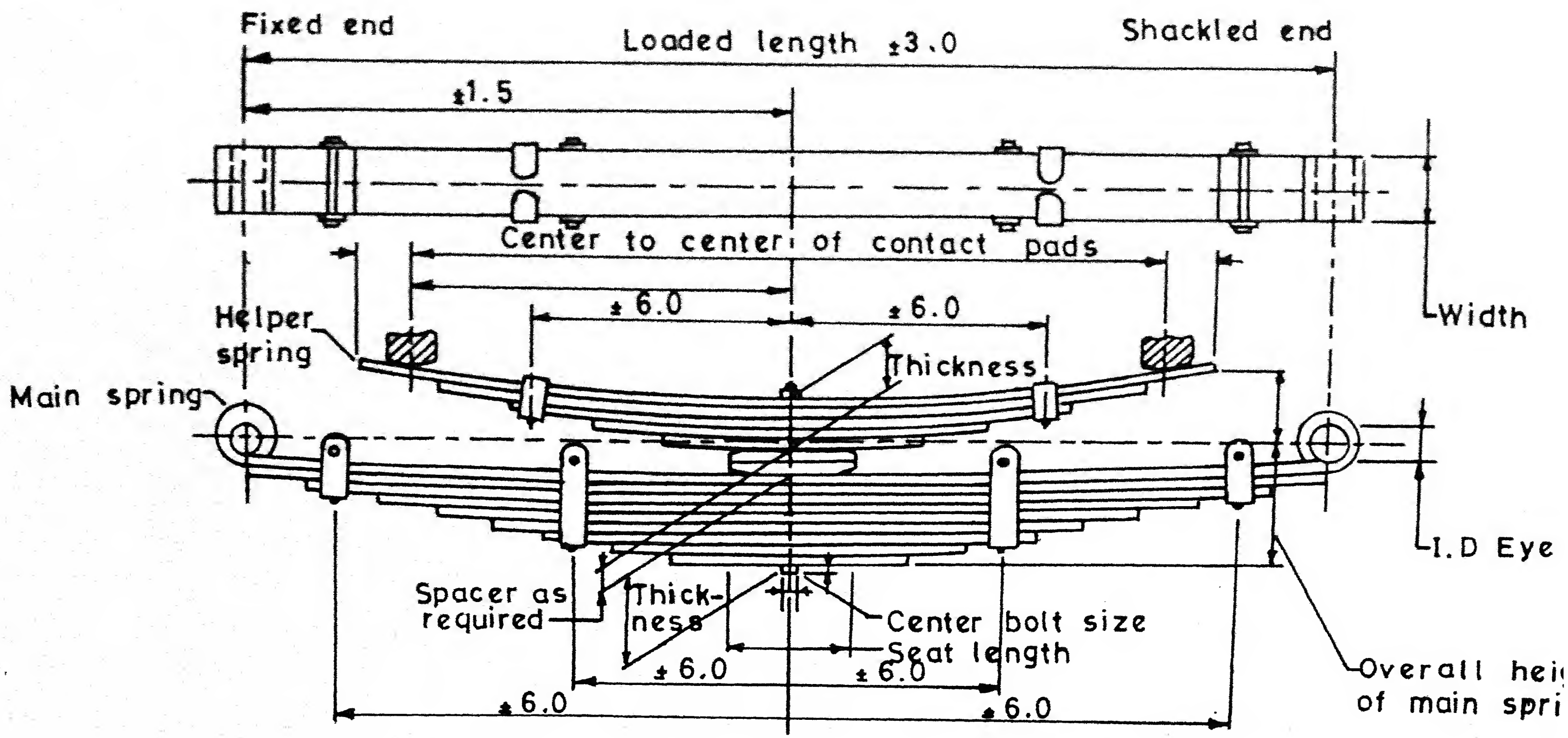
Dimensions indicate tolerance in mm.

Fig. I.3 Spring with one eye and one plain end.




Dimensions indicate tolerance in mm.

Fig. I.4 Underslung spring.




Dimensions indicate tolerance in mm.

Fig. I.5 Overslung spring.

## APPENDIX-II

Center Link Extension Method - Procedure (Fig. B-1):

1. Start layout with main leaf in flat position with lengths a, b and L measured along the main leaf and axle center H, at distance h from center of main leaf. Axle center is above main leaf in an underslung spring, below main leaf in an overslung spring.

2. Establish clamp lengths m and n, which represent the inactive material. These are considered equal for most springs. They can be neglected in relatively long flexible springs without serious error.

3. Draw arc $R_a$ and at the intersection with $0.5\ e_a$ locate point D.

4. Draw arc $R_b$ and at the intersection with $0.5\ e_f$ locate point E.

5. Construct the three links AD, DE and EB.

6. Locate point M at the intersection of center line of center bolt and link DE.

7. Locate point O on extension of center link DE at computed distance Q from point M (Table II-1)

8. Draw arc $R_M$ , where $R_M = \lambda \cdot L$. Its center is located on extension of line OA.

9. For a given deflection in rebound or compression, new position of Center link DE is established by locating point $M_r$ or $M_c$ and then drawing a line through point $M_r$ or $M_c$ and point O.

10. For each position of the center link DE, the axle position can be located by constructing the triangle DEH. When three or more such positions have been located, the approximate radius $R_H$ of the axle can be established by geometric construction.

11. The control or tilt of the center link - and thus of the spring seat - in degrees per mm is equal to the angle $\theta$ divided by the deflection x.

12. In the symmetrical spring control is zero, with the center link moving parallel to itself throughout the compression and rebound region. Actually, however, the center link undergoes a smaller angular change due to the vertical displacement of the shackled spring eye.

13. Depending upon the accuracy demanded of the layout, a correction for the effect of the shakle may be necessary, particularly when the shackle

angle is exceptionally small ($\beta$ less than 60° in the flat main leaf position) and the shackle is exceptionally long, the correction may be made in the following manner:

a) Locate point P at intersection of datum line and $R_o$ , where $R_o$ is equal to distance OA.

b) After determining linkage layout for a given deflection, such as for rebound, point $B_r$ is located.

c) Locate point $P_r$ on extension of line $B_rA$.

d) Locate point $O_r$ where chordal distance Po equals $P_r$ $O_r$.

e) Draw line $O_r$ $M_r$ to give corrected tilt to center link DE in rebound.

f) In like manner, establish line $O_c$ $M_c$ to give corrected tilt to center link DE in compression.

g) These corrected positions of center link DE determine the corrected control in degrees per mm (equal to $\theta'/x$) and can be used to establish the approximate radius $R'_H$ for the corrected axle path.

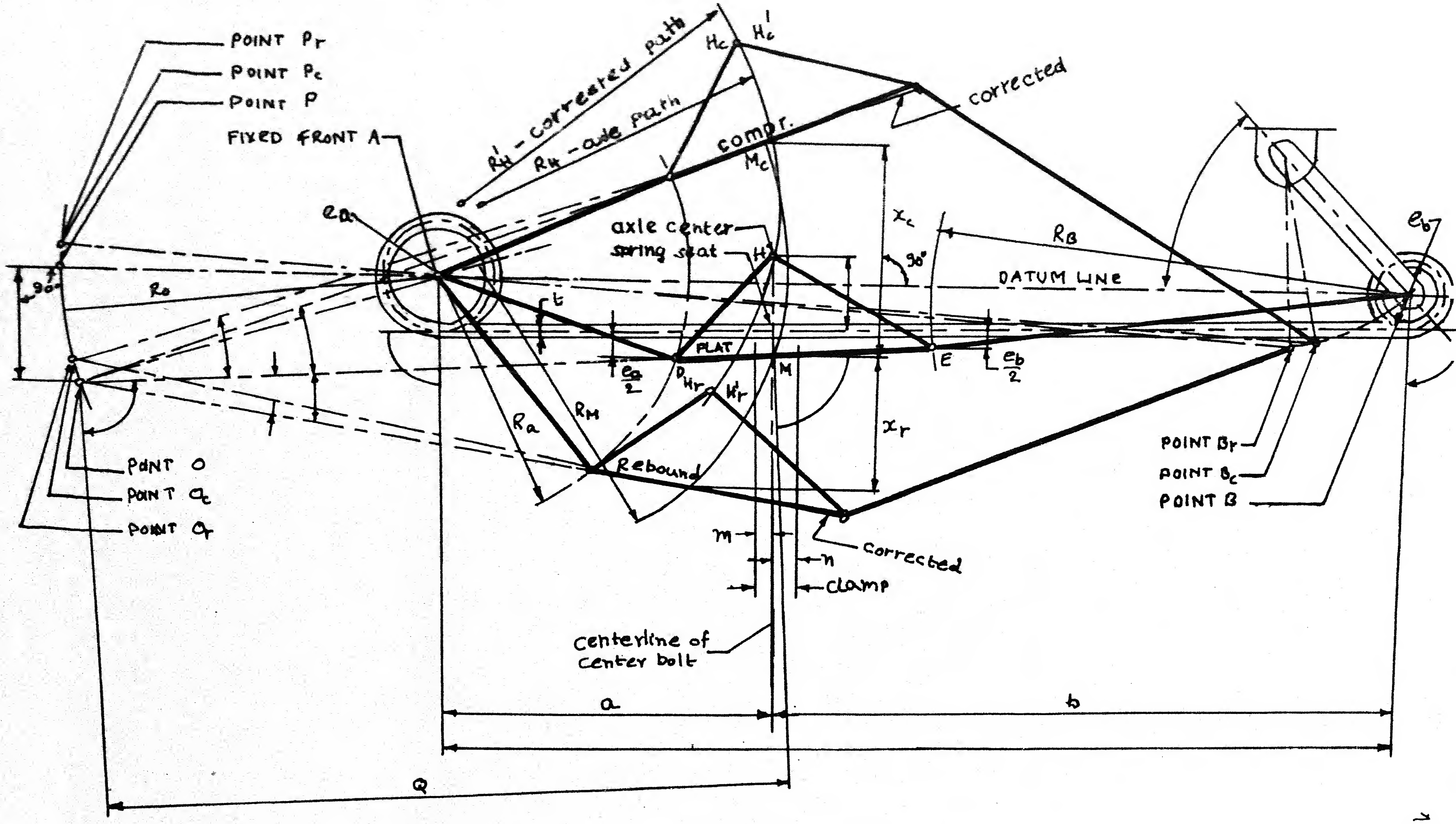



Fig.II.1 Layout by center link extension method using 3-link mechanism.

## TABLE-IV-1

### Geometry Formulae for Semi-Elliptic Springs

A. For conventional spring where $W = Z/Y^3$ equals one

$$f = x/Y \qquad g = x.Y = f.Y^2$$

$$q = \frac{3(b.e_a + a.e_b) + 6Q\,(e_a - e_b)}{2L} \qquad Q = \frac{a.b}{b-a} = \frac{L.Y}{(Y^2-1)}$$

$$\lambda = \frac{3Y^2}{(3Y^2 + 1)(Y + 1)} \qquad V = \frac{Y^2}{Y + 1}$$

$$Z = \frac{b(57.3 + \phi.b)}{a(57.3 - \phi.a)} = Y \cdot \frac{57.3(Y + 1) + \phi.L.Y}{57.3(Y + 1) - \phi.L}$$

$$\phi = \frac{57.3}{Q} = \frac{57.3(b - a)}{a \cdot b} = \frac{57.3(Y^2 - 1)}{L \cdot Y}$$

B. For unconventional spring where $W = Z/Y^3$ does not equal one

$$f = x \cdot \frac{Y(Y+1)}{Z+Y.Y} \qquad g = x.\frac{Z(Y+1)}{Z+Y^2} = f.\frac{Z}{Y}$$

q = Same as in A.

$$Q = \frac{Z.a^2+b^2}{Z.a-b} = \frac{L(Z+Y^2)}{(Z-Y)(Y+1)}$$

$$\lambda = \frac{3(Z+Y^2)^2}{3(Z+Y^2) + Y^2(Y+1)^2} \cdot \frac{1}{Y+1} \qquad V = \frac{Z+Y^2}{(Y+1)^2}$$

Z = Same as in A.

$$\phi = \frac{57.3}{Q} = \frac{57.3(Z.a-b)}{Z.a^2+b^2} = \frac{57.3(Z-Y)(Y+1)}{L(Z+Y^2)}$$

# APPENDIX-III

## Two Point Deflection Method - Procedure (Fig. C-1)

1. Start layout with main leaf in flat position with length a, b and L measured along the main leaf and axle center H at distance h from center of main leaf.

2. Establish clamp lengths m and n which represent the inactive material.

3. Draw arc $R_a$ and at the intersection with $0.5e_a$ locate point D.

4. Draw arc $R_b$ and at the intersection with $0.5e_b$ locate point E.

5. Construct the three links AD, DE, and EB.

6. Locate point M at intersection of center line of center bolt and link DE.

7. Draw reference lines AF and BG through the eye centers and perpendicular to the extension of the center link DE.

8. For any given deflections such as $x_r$ and $x_c$, compute $f_r$ and $f_c$ from the following formulae, and draw arcs about point F.

a) for a conventional spring,

$$f = \frac{x}{Y} = x \ (a/b)$$

b) for an unconventional spring,

$$f = x \cdot \frac{Y\ (1 + Y)}{Z + Y^2}$$

9) Similarly, for given deflections $x_r$ and $x_c$, compute $g_r$ and $g_c$ from the following formulae, and draw arcs about point G.

a) for a conventional spring,

$$g = x\,Y = x\ (b/a)$$

b) for an unconventional spring,

$$g = f\ (Z/Y)$$

10. Tangent lines to arc $f_r$ and $g_r$ establishes the position of centerlink DE in rebound, and tangent line to arcs $f_c$ and $g_c$ establishes the position of center link DE in compression.

11. For each position of the center link DE the axle position can be located by the triangle DEH when three or more such positions have been located, the approximate radius $R_H$ for the axle path can be established by geometric construction.

12. The control in degrees per mm is equal to the angular change in position of the center link DE divided by the deflection x.

13. In the symmetrical spring the control is zero, with the center link moving parallel to itself throughout the compression and rebound range. Actually, however, the center link undergoes a small angular change due to the vertical displacement of the shackled spring eye.

14. Depending upon the accuracy demanded of the layout, a correction for the effect of the shackle may be necessary, particularly when the shackle angle is exceptionally small ($\beta$ less than 60° in the flat main leaf position) and the shackle is exceptionally long. The correction may be made in the following manner:

a) After determining linkage layout for a given deflection, such as for rebound, point $B_r$ is located.

b) Draw arc $d_r$ where $d_r = (g_r - u)$

c) Draw tangent line to arcs $f_r$ and $d_r$ to establish corrected position of center link DE in rebound.

d) Similarly after locating $B_c$ draw arc $d_c$ where $d_c = (g_r + u)$

e) Draw tangent line to arcs $f_c$ and $d_c$ to establish corrected position of center link DE in compression

f) These corrected positions of center link DE determine the corrected control in degrees per mm and can be used to establish the approximate radius $R'_H$ for the corrected axle path.

g) The difference between deflections $x_c$, $x_r$ and $x'_c$, $x'_r$ respectively is so small in a full size spring layout that it can usually be neglected.

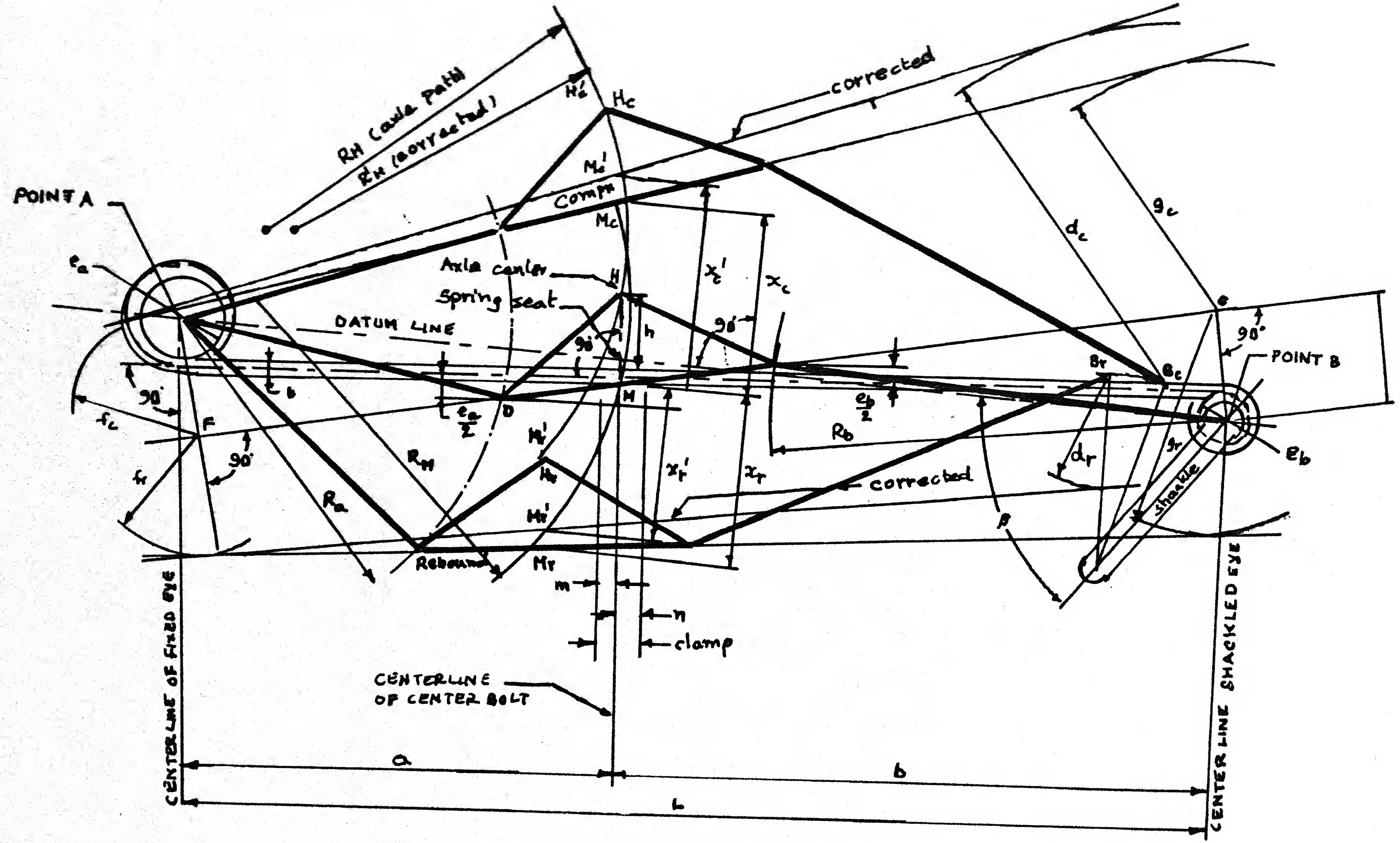



Fig. III.1 Layout by two point deflection method.

APPENDIX IV-1

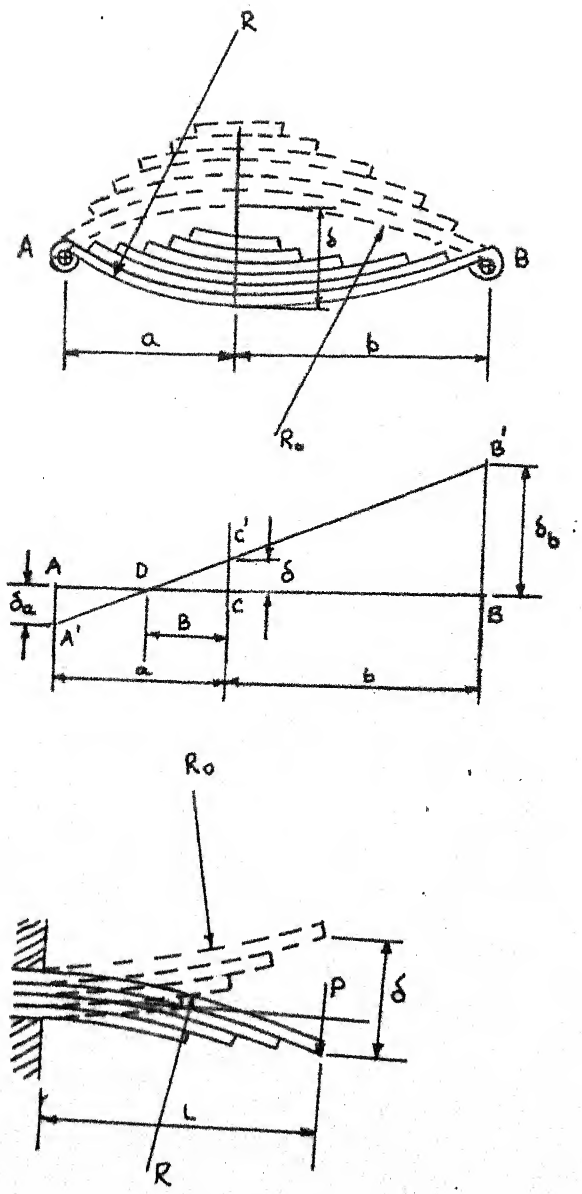



Fig.IV.1 Unsymmetric semi-elliptic leaf spring.

# APPENDIX-IV

## UNSYMMETRICAL SEMI-ELLIPTIC SPRING

Deflection: (Fig. IV-1)

Spring has N leaves, width w and thickness t.

Unloaded springs have radius which is considered negative.

If $\delta_a$ - deflection of front cantilever

$\delta_b$ - deflection of rear cantilever,

from similar triangles DCC' and DBB',

$$\frac{\delta_b}{\delta} = \frac{B+b}{b} = 1 + \frac{b}{B}$$

$$\frac{b}{B} = \frac{\delta_b}{\delta} - 1 = \frac{\delta_b - \delta}{\delta}$$

$$B = \frac{b.\delta}{\delta_b - \delta}$$

from similar triangles DCC' and AA'D

$$\frac{-\delta_a}{\delta} = \frac{a-B}{B} = \frac{a}{B} - 1$$

$$\frac{\delta_a}{\delta} = 1 - \frac{a}{B} = 1 - a . \frac{(\delta_b - \delta)}{b.\delta}$$

$$\frac{\delta_a}{\delta} = \frac{b.\delta - a.\delta_b + a.\delta}{b.\delta} = \frac{(a+b)\delta - a.\delta_b}{b.\delta}$$

$$b\,\delta_a = L.\delta - a\,\delta_b$$

$$\delta = \left(\frac{a.\delta_b + b.\delta_a}{L}\right)$$

Consider a uniform strength cantilever beam,

from geometry,

$$\delta = \frac{L^2}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)$$

A semi-elliptic spring can be considered as two cantilevers with deflections $\delta_a$ and $\delta_b$ and combined by the equation

$$\delta = \frac{a.\delta_b + b.\delta_a}{L}$$

For front cantilever,

$$\delta_a = \frac{a^2}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)$$

and rear cantilever,

$$\delta_b = \frac{b^2}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)$$

$$\delta = \frac{a.\frac{b^2}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right) + b.\frac{a^2}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)}{L}$$

$$= \frac{ab}{2L}\left( \frac{1}{R} - \frac{1}{R_o} \right)(b + a) \qquad \because \underline{a + b = L}$$

$$\delta = \frac{ab}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)$$

Stress from strain is given by

$$\sigma = \frac{Et}{2}\left( \frac{1}{R} - \frac{1}{R_o} \right)$$

From the deflection formula

$$\left( \frac{1}{R} - \frac{1}{R_o} \right) = \frac{2\delta}{ab}$$

$$\therefore \quad \sigma = \frac{Et}{2} \cdot \frac{2\delta}{ab} \cdot SF$$

$$\sigma = \frac{Et}{ab} \cdot \delta \cdot SF$$

A stiffness factor SF is used to calculate the stress from deflection

$$SF = 1 + \frac{n}{N}$$

n = number of full length leaves

N = total number of leaves.

Consider the B M Equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

$$\sigma = \frac{MY}{I}$$

M = Maximum bending moment

$$= \frac{P.a.b}{L}$$

y = t/2

t = thickness of leaf

w = width

I = moment of inertia $= w \cdot \frac{N\,t^3}{12}$

$$\sigma = \frac{P \cdot a \cdot b}{L} \cdot \frac{t}{2} \cdot \frac{1}{I}$$

$$\therefore \quad = \frac{P.a.b.t}{2IL}$$

$$= \frac{a\ b}{2 . \frac{wNt^3}{12}} \cdot \frac{Pt}{L}$$

$$\sigma = \frac{6ab}{wNt^3} \cdot \frac{Pt}{L}$$

The stiffness of the spring, K is

$$K = \frac{P}{\delta}$$

$$\sigma = \frac{Et}{ab} \cdot \delta \cdot SF = \frac{6ab}{wNt^3} \cdot \frac{Pt}{L}$$

$$\therefore \quad \frac{P}{\delta} = \frac{Et}{ab} \cdot SF \cdot \frac{wNt^3}{6ab} \cdot \frac{L}{t}$$

$$K = \frac{E}{6} \cdot \frac{wNt^3}{a^2b^2} \cdot SF$$

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